basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11
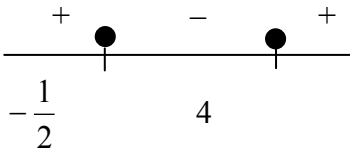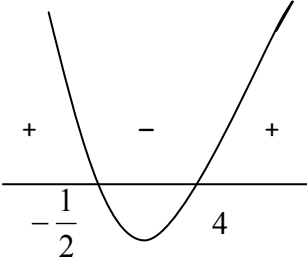
MATHEMATICS P1

NOVEMBER 2014

MEMORANDUM

MARKS: 150

This memorandum consists of 14 pages.

QUESTION 1

1.1.1	$x = -2 \ \text{ or } \ x = \dfrac{7}{3}$	$\checkmark x = -2$ $\checkmark x = \dfrac{7}{3}$ (2)
1.1.2	$x^2 - 5x - 2 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{5 \pm \sqrt{25 - 4(1)(-2)}}{2}$ $x = \dfrac{5 \pm \sqrt{33}}{2}$ $x = 5{,}37 \quad \text{or} \quad x = -0{,}37$ **OR** $x^2 - 5x + \left(\dfrac{25}{4}\right) = 2 + \left(\dfrac{25}{4}\right)$ $\left(x - \dfrac{5}{2}\right)^2 = \dfrac{33}{4}$ $x - \dfrac{5}{2} = \pm \dfrac{\sqrt{33}}{2}$ $x = \dfrac{5 + \sqrt{33}}{2} \quad \text{or} \quad x = \dfrac{5 - \sqrt{33}}{2}$ $x = 5{,}37 \qquad\qquad x = -0{,}37$	$\checkmark$ standard form $\checkmark$ correct substitution into correct formula $\checkmark x = 5{,}37$ $\checkmark x = -0{,}37$ (4) $\checkmark$ completing the square $\checkmark \sqrt{33}$ $\checkmark x = 5{,}37$ $\checkmark x = -0{,}37$ (4)
1.1.3	$\sqrt{x - 3} = 5 + 4$ $\left(\sqrt{x - 3}\right)^2 = (9)^2$ $x - 3 = 81$ $x = 84$	$\checkmark$ isolating $\sqrt{}$ $\checkmark$ squaring both sides $\checkmark$ simplify $\checkmark$ answer (4)
1.1.4	$2x^2 - 7x - 4 \geq 0$ $(2x + 1)(x - 4) \geq 0$ CV's : $-\dfrac{1}{2}; 4$ $x \leq -\dfrac{1}{2} \quad \text{or} \quad x \geq 4$ **OR** $x \in (-\infty; -\dfrac{1}{2}] \cup [4; \infty)$	$\checkmark$ factors $\checkmark$ method $\checkmark$ notation $\checkmark$ critical values (4) $\checkmark$ notation $\checkmark$ critical values

| 1.2 | $x = 2y + 1$ ......(1)
 $x^2 - 2y + 3xy = 6$ ....(2)
 $(2y+1)^2 - 2y + 3y(2y+1) = 6$
 $4y^2 + 4y + 1 - 2y + 6y^2 + 3y - 6 = 0$
 $10y^2 + 5y - 5 = 0$
 $2y^2 + y - 1 = 0$
 $(2y-1)(y+1) = 0$
 $y = \dfrac{1}{2}$ or $y = -1$
 $x = 2$ $x = -1$

 OR
 $y = \dfrac{x-1}{2}$
 $x^2 - 2\left(\dfrac{x-1}{2}\right) + 3x\left(\dfrac{x-1}{2}\right) = 6$
 $2x^2 - 2x + 2 + 3x^2 - 3x - 12 = 0$
 $5x^2 - 5x - 10 = 0$
 $x^2 - x - 2 = 0$
 $(x+1)(x-2) = 0$
 $x = -1$ or $x = 2$
 $y = -1$ $y = \dfrac{1}{2}$ | ✓ substitution of $x = 2y+1$
 ✓ simplification

 ✓ standard form
 ✓ factors

 ✓ both y values
 ✓ both x values   (6)

 ✓ substitution of $y = \dfrac{x-1}{2}$
 ✓ simplification

 ✓ standard form

 ✓ factors
 ✓ both x values
 ✓ both y values   (6)

 [20] |

QUESTION 2

2.1	$\dfrac{3^x(3-3^{-1})}{2.3^x}$	✓ common factor 3^x ✓ $3-3^{-1}$
	$=\dfrac{3-\dfrac{1}{3}}{2}$	✓ answer (3)
	$=\dfrac{8}{3}\times\dfrac{1}{2}$	✓ common factor 3^{x-1}
	$=\dfrac{4}{3}$	✓ simplification
	OR	✓ answer (3)
	$\dfrac{3^{x-1}(3^2-1)}{2.3^x}$	
	$=\dfrac{3^x.3^{-1}(8)}{2.3^x}$	
	$=\dfrac{1}{3}\times 4$	
	$=\dfrac{4}{3}$	
2.2	$(x-2)^{-\frac{3}{2}}=64$	
	$x-2=\left[(4^3)\right]^{\frac{-2}{3}}$	✓ applying exp. law ✓ 4^3 ✓ simplifying
	$x-2=4^{-2}$	
	$x=2+\dfrac{1}{16}$	✓ answer (4)
	$\therefore x=2\dfrac{1}{16}$	
	OR	
	$\sqrt{(x-3)^{-3}}=64$	
	$(x-3)^{-3}=4096$	✓ squaring
	$(x-2)^3=\dfrac{1}{4096}$	✓ applying exp. law
	$x-2=\dfrac{1}{16}$	✓ simplification
	$x=2\dfrac{1}{16}$	✓ answer (4)

| 2.3 | $\dfrac{x.x^{\frac{1}{2}}.x^{\frac{1}{4}}.x^{\frac{1}{8}}}{\sqrt[8]{x^7}}$ $=\dfrac{x^{\frac{7}{8}}}{x^{\frac{7}{8}}}$ $=x$ | ✓applying surd law ✓applying surd law ✓simplifying ✓answer (4) **[11]** |

QUESTION 3

| 3 | $AC.(x-2)=x^2+2x-8$ $AC.(x-2)=(x+4)(x-2)$ $AC=(x+4)\text{ cm}$ $\therefore FD=(x+4)\text{ cm}$ $\therefore ED=x+4-(x-2)$ $ED=6\text{ cm}$ | ✓ statement ✓ factors ✓ $AC=(x+4)$ cm ✓ method ✓ answer (6) **[6]** |
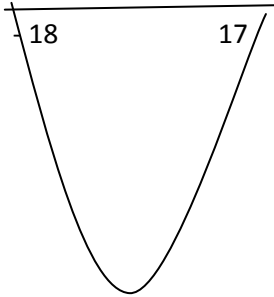
QUESTION 4

| 4.1 | $-7\quad 0\quad 9\quad 20$ $\quad 7\quad 9\quad 11$ $\quad\quad 2\quad 2$ $2a=2$ $a=1$ $3(1)+b=7$ $b=4$ $(1)+(4)+c=-7$ $c=-12$ $\therefore T_n=n^2+4n-12$ **OR** $2a=2$ $a=1$ $T_2=2^2+b(2)+c=0$ $\quad 2b+c=-4\quad (1)$ $T_3=3^2+b(3)+c=9$ $\quad 3b+c=0\quad (2)$ $(2)-(1)\quad b=4$ $\therefore c=-4-2(4)=-12$ $T_n=n^2+4n-12$ | ✓ $2a=2$ ✓$a$value ✓$b$value ✓c value (4) ✓ $2a=2$ ✓$a$value OR $3(1)+b=7$ $b=4$ $1+a+c=-7$ $c=-12$ ✓$b$value ✓c value (4) |

	OR $$T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}.d_2$$ $$= -7 + (n-1).7 + \frac{(n-1)(n-2)}{2}.2$$ $$= -7 + 7n - 7 + n^2 - 3n + 2$$ $$= n^2 + 4n - 12$$	✓formula ✓✓substitution ✓simplification (4)
4.2	$$n^2 + 4n - 12 = 128$$ $$n^2 + 4n - 140 = 0$$ $$(n+14)(n-10) = 0$$ $$n \neq -14 \text{ or } n = 10$$ invalid $\therefore n = 10$	✓ equation ✓factors ✓answers for n ✓$n = 10$ (choice) (4)
4.3	$-7 ; 0 ; 9 ; 20 ; \ldots$ first difference 7 9 11 second difference 2 2 $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$	✓first differences Answer only: Full Marks ✓$c = 5$ (3)
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ this difference will be between term 297 and term 298	✓ equating ✓297 ✓298(3) **[14]**

QUESTION 5

5.1					
	Pattern	1	2	3	
	White squares	4	12	24	
	40				✓✓answer(2)
5.2	$W_n = 2n^2 + 2n$ $W_{157} = 2(157)^2 + 2(157)$ $= 49612$				✓W_n ✓substitution ✓ answer (3)

Please turn over

5.3	$2n^2 + 2n + 1 < 613$ $2n^2 + 2n - 612 < 0$ $n^2 + n - 306 < 0$ $(n - 17)(n + 18) < 0$ $\therefore n = 16$	✓setting up inequality ✓factors ✓method ✓answer (4)

5.4	$P_n = 4n^2 + 4n + 1$
	$= (2n)^2 + 2(2n) + 1$
	$2n$ is even for all $n \in Z$
	$\therefore$ Total squares used in the nth pattern is always odd.
	OR
	$P_n = 4n^2 + 4n + 1$
	$= 2(2n^2 + 2n) + 1$
	$2(2n^2 + 2n)$ is odd for all $n \in Z$
	$2(2n^2 + 2n) + 1$ is odd for all $n \in Z$
	$\therefore$ Total squares used in the nth pattern is always odd.
		✓ $P_n = 4n^2 + 4n + 1$
		✓rewriting P_n
		
		✓conclusion
		    (3)
		
		✓ $P_n = 4n^2 + 4n + 1$
		✓ rewriting P_n
		
		✓ conclusion
		    (3)
		
		[12]

QUESTION 6

6.1	$x = 2$ $y = 3$	✓ $x = 2$ ✓ $y = 3$ (2)
6.2	$x.\text{int} : \dfrac{8}{x-2} + 3 = 0$ $8 + 3(x - 2) = 0$ $3x + 2 = 0$ $\therefore x = -\dfrac{2}{3}$ $\therefore x - \text{int}\left(-\dfrac{2}{3}; 0\right)$ $y = \dfrac{8}{0-2} + 3$ $y = -1$ $y.\text{int} : (0; -1)$	✓ $\dfrac{8}{x-2} + 3 = 0$ ✓ $\left(-\dfrac{2}{3}; 0\right)$ ✓ $(0; -1)$ (3)

6.3		✓asymptotes ✓intercepts with axes ✓shape (3)
6.4	$3 = 2 + k$ $k = 1$ **OR** $y = (x - 2) + 3$ $y = x + 1$ $\therefore \quad k = 1$	✓ substitute ✓ answer (2) ✓ $y = x + 1$ ✓answer (2) **[10]**

QUESTION 7

7.1	$q = -6$	✓answer (1)
7.2	$-5\dfrac{1}{4} = a.2^{-1-1} - 6$ $\dfrac{3}{4} = \dfrac{1}{4}a$ $a = 3$	✓substitute x ✓substitute y ✓simplifying ✓answer (4)
7.3	$x\,\text{int}: 2^{x-1} = 2 \quad \therefore x = 2 \therefore (2;0)$ $y\,\text{int}: y = 3.2^{-1} - 6 = -4\dfrac{1}{2} \quad \therefore \left(0; -4\dfrac{1}{2}\right)$ Average Gradient $= \dfrac{0 + 4\dfrac{1}{2}}{2 - 0}$ $= \dfrac{9}{4} \text{ or } 2\dfrac{1}{4}$	✓ $2^{x-1} = 2$ ✓ $x = 2$ ✓$y = -4\dfrac{1}{2}$ ✓subst. into gradient formula ✓answer (5)
7.4	$y = 3.2^{x-3} - 6$	✓✓answer (2) **[12]**

QUESTION 8

8.1	C(-1 ; 0)	✓ C(-1 ; 0) (1)
8.2	$y = (x-3)(x+1)$ $y = x^2 - 2x - 3$	✓ $(x-3)$ ✓ $(x+1)$ ✓ $y = x^2 - 2x - 3$ (3)
8.3	TP : $y = (1)^2 - 2(1) - 3$ $\quad y = -4$ R: $y \in [-4;\infty)$ **OR** $y \geq -4$	✓ $y = -4$ ✓ $[-4;\infty)$ (2) ✓ $y \geq -4$
8.4	$m = \dfrac{0+4}{3-1} = 2$ $y - 0 = 2(x-3)$ $y = 2x - 6$	✓ substituting into gradient formula ✓ $m = 2$ ✓ equation (3)
8.5.1	$x \leq -1 \quad$ or $\quad x \geq 3$ **OR** $x \in (-\infty;-1] \cup [3;\infty)$	✓ $x \leq -1$ ✓ $x \geq 3$ (2) ✓ $(-\infty;-1]$ ✓ $[3;\infty)$ (2)
8.5.2	$-1 < x < 3 \quad$ or $\quad x > 3$ **OR** $x > -1 \quad ; x \neq 3$ **OR** $(-1;3) \cup (3;\infty)$	✓ critical values ✓ notation (2) ✓ $x > -1$ ✓ $x \neq 3$ (2) ✓ $(-1;3)$ ✓ $(3;\infty)$ (2)
8.5.3	$-1 < x < 0 \quad$ or $\quad x > 3$ **OR** $(-1;0) \cup (3;\infty)$	✓ critical values ✓ notation (2) ✓ $(-1;0)$ ✓ $(3;\infty)$ (2)

8.6	$x^2 - 2x - p = 0$	
	$\Delta = (-2)^2 - 4(1)(-p)$	
	$\quad = 4 + 4p$	
	for non - real roots $\Delta < 0$	
	$4 + 4p < 0$	✓ $4 + 4p < 0$
	$4p < -4$	
	$\therefore p < -1$	✓ $p < -1$ (2)
	OR	
	A$(1; -4)$	
	$x^2 - 2x - 3 = 0$	
	$x^2 - 2x - p = 0$	✓ $-p > 1$
	$-p > 1$	✓ $p < -1$ (2)
	$\therefore p < -1$	
8.7	$PM = (2x - 6) - (x^2 - 2x - 3)$	✓ subtraction
	$\quad = -x^2 + 4x - 3$	✓ quadratic expression
	$x = -\dfrac{b}{2a}$	✓ method
	$\quad = -\dfrac{4}{2(-1)} = 2$	
	$Max.PM = -(2)^2 + 4(2) - 3 = 1$ unit	✓ maximum value (4)
	OR	
	$PM = (2x - 6) - (x^2 - 2x - 3)$	✓ subtraction
	$\quad = -x^2 + 4x - 3$	✓ quadratic expression
	$\quad = -(x^2 - 4x + 4 - 4 + 3)$	✓ method
	$\quad = -[(x - 2)^2 - 1]$	
	$\quad = -(x - 2)^2 + 1$	
	$Max.PM = 1$ unit	✓ maximum value (4)
		[21]

QUESTION 9

9.1	$A = P(1 - i)^n$	✓ substitution
	$11090,41 = 120000(1 - i)^{12}$	
	$\therefore i = 1 - \sqrt[12]{\dfrac{11090,41}{120000}}$	✓ making i subject
		✓ i value as decimal
	Thus $i = 0,179999...$	
	Rate of Depreciation $= 18\%$	✓ answer (4)

9.2	$i_{eff} = \left(1 + \dfrac{i}{m}\right)^m - 1$ $= \left(1 + \dfrac{0,098}{12}\right)^{12} - 1$ $= 0,10252.....$ rate $= 10,25\%$	✓formula ✓substitution into formula ✓10,25% (3)
9.3	$A = P(1 + i_1)^{n_1}(1 + i_2)^{n_2}$ $= 80\,000\left(1 + \dfrac{0.075}{4}\right)^{16}\left(1 + \dfrac{0,092}{12}\right)^{36}$ $= R141\,768,60$ **OR** $A_1 = 80\,000\left(1 + \dfrac{0,075}{4}\right)^{16}$ $= 107\,689,1465..$ $A_2 = 107\,689,1465\left(1 + \dfrac{0,092}{12}\right)^{36}$ $= R141\,768,60$	✓$\left(1 + \dfrac{0.075}{4}\right)^{16}$ ✓$\left(1 + \dfrac{0,092}{12}\right)^{36}$ ✓multiplication ✓answer (4) ✓$\left(1 + \dfrac{0,075}{4}\right)^{16}$ ✓A_1 ✓$\left(1 + \dfrac{0,092}{12}\right)^{36}$ ✓answer (4)
9.4.1	Investment : end of third year : $A = P(1 + i)^n$ $= 30\,000\left(1 + \dfrac{0,065}{12}\right)^{96}$ $= R50\,390,07$	✓$\dfrac{0,065}{12}$ ✓subst. into correct formula ✓answer (3)
9.4.2	$\begin{array}{cccc} T_0 & T_3 & T_5 & T_8 \\ \hline 30000 & -10000 & +10000 & \end{array}$ $A = 30000\left(1 + \dfrac{0,65}{12}\right)^{96} - 10000\left(1 + \dfrac{0,65}{12}\right)^{60} + 10000\left(1 + \dfrac{0,65}{12}\right)^{36}$ $A = R48\,708,61$ $\therefore$ difference $= 48\,708,61 - 50\,390,07$ $\qquad\qquad = -R1\,681,46$	✓$30000\left(1 + \dfrac{0,65}{12}\right)^{96}$ ✓$-10000\left(1 + \dfrac{0,65}{12}\right)^{60}$ ✓$10000\left(1 + \dfrac{0,65}{12}\right)^{36}$ ✓R48 708,61 ✓subtracting ✓answer (7)

| | Investment : end of third year : $A = P(1+i)^n$ $= 30\,000\left(1+\dfrac{0,065}{12}\right)^{36}$ $= R36\,440,14881$ Principal(new) : $R36\,440,14881 - R10\,000,00 = R26\,440,14881$ Investment : end of fifth year : $A = P(1+i)^n$ $= 26\,440.14881\left(1+\dfrac{0,065}{12}\right)^{24}$ $= R30\,100,2304$ Principal(new) : $R30\,100,2304 + R10\,000,00 = R40\,100,2304$ Investment : end of eighth year : $A = P(1+i)^n$ $= 40\,100,2304\left(1+\dfrac{0,065}{12}\right)^{24}$ $= R48\,708,61$ Tashil had a deficit of R1681,46 | ✓subst. into formula ✓answer ✓subst. into formula ✓answer ✓subst. into formula ✓answer ✓conclusion(7) **[21]** |

QUESTION 10

10.1	5 customers	✓answer (1)
10.2	P(C and B) $\neq$ 0 Thus events B and C are not mutually exclusive	✓P(C and B) $\neq$ 0 ✓ conclusion (2)
10.3.1	P(V only) $= \dfrac{58}{240} = \dfrac{29}{120}$	✓ answer (1)
10.3.2	P(C and B) $= \dfrac{29}{240}$	✓ answer (1)
10.3.3	P(not C) = 1 – P(C) $= 1 - \dfrac{122}{240} = \dfrac{59}{120}$ **OR** P(not C) $= \dfrac{52 + 3 + 58 + 5}{240}$ $= \dfrac{118}{240} = \dfrac{59}{120}$	✓ formula ✓ substitution ✓answer (3) ✓✓numerator and denominator ✓answer (3)

10.3.4	P(B or V) = P(B) + P(V) – P(B and V) $= \dfrac{84}{240} + \dfrac{82}{240} - \dfrac{15}{240}$ $= \dfrac{151}{240}$ **OR** $P(\text{B or V}) = \dfrac{17 + 52 + 12 + 3 + 9 + 58}{240}$ $= \dfrac{151}{240}$	$\checkmark \dfrac{84}{240}$ $\checkmark \dfrac{82}{240}$ $\checkmark \dfrac{15}{240}$ $\checkmark \dfrac{151}{240}$ (4) $\checkmark\checkmark$ numerator and denominator $\checkmark\checkmark$ answer (4) **[12]**

QUESTION 11

	$P(\text{A or B}) = P(A) + P(B) - P(A \cap B)$ $0{,}428 = 0{,}12 + 0{,}35 - P(A \cap B)$ $P(A \cap B) = 0{,}042$ $P(A) \times P(B) = 0{,}12 \times 0{,}35 = 0{,}042$ $\therefore P(A \cap B) = P(A) \times P(B)$ Thus A and B are independent events	$\checkmark$ substitution $\checkmark$ value of $P(A \cap B)$ $\checkmark$ value of $P(A) \times P(B)$ $\checkmark$ conclusion (4) **[4]**

QUESTION 12

12.1	There are 100% – 60% – 10% = 30% red marbles $$\therefore \frac{30}{100} \times 80 = 24 \text{ red marbles}$$	✓30% ✓24 (2)
12.2		✓first branch ✓second branch ✓values on diagram (3)
12.3	P(G and Y) = P(G, Y) + P(Y, G) $$= \frac{48}{80} \times \frac{8}{79} + \frac{8}{80} \times \frac{48}{79}$$ $$= \frac{48}{395}$$	✓ multiplication rule ✓ addition ✓answer (3) **[8]**
		TOTAL: 150