Basic Trigonometric Relationships - Reduction Formulae and Identities

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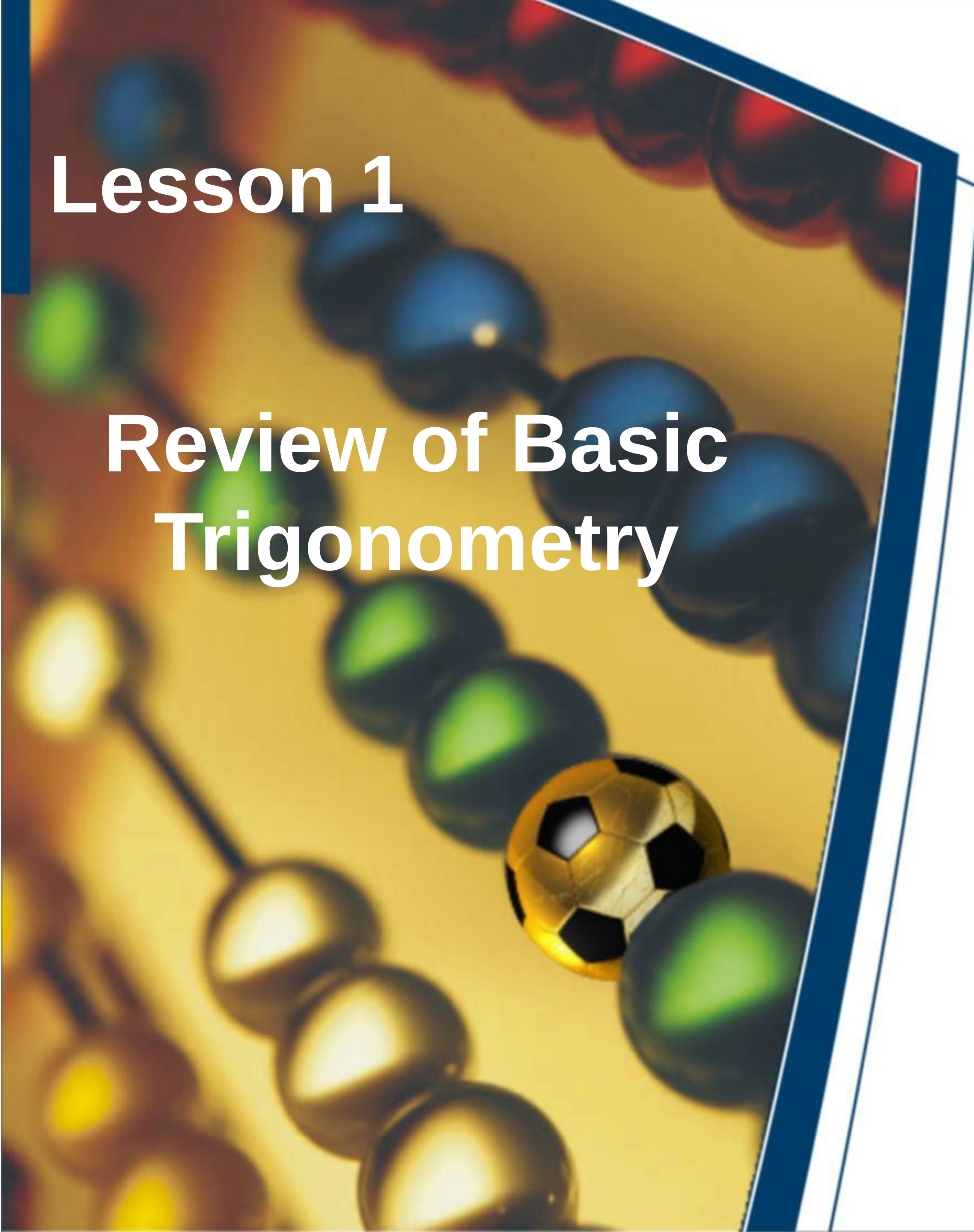
Outcomes for this DVD

In this DVD we will:

- Review basic trigonometric relationships LESSON 1
- Work with reduction formulae in respect of the angles $-\theta,\ 180° \pm \theta,\ 90° \pm \theta$ and $360° \pm \theta$. LESSON 2
- Learn about and apply basic trigonometric identities. LESSON 3

Lesson 1

Review of Basic Trigonometry



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Review: Definition of the Trigonometric Functions

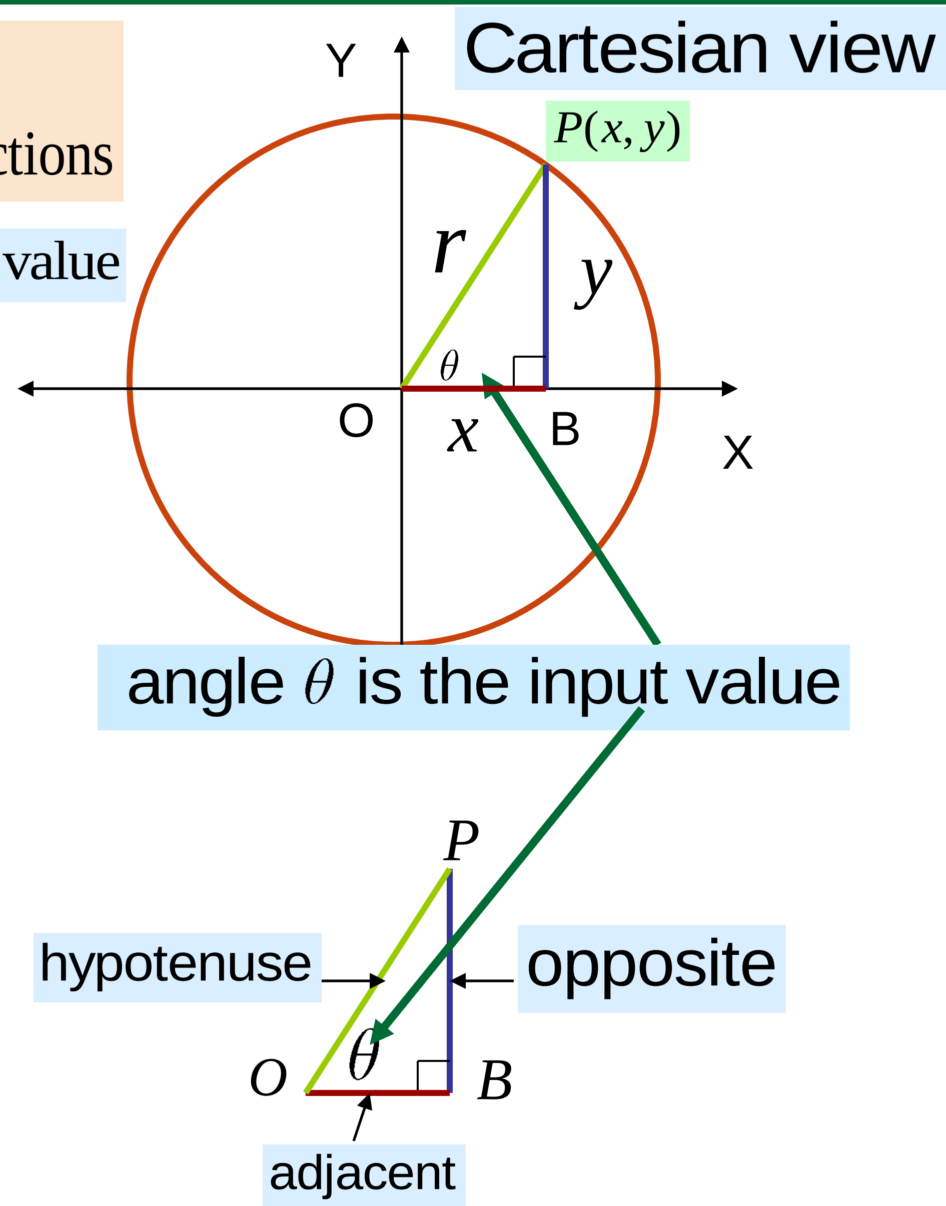
Ratios of the sides of a right triangle determines the value of trigonometric functions

Trigonometric function of θ gives output value

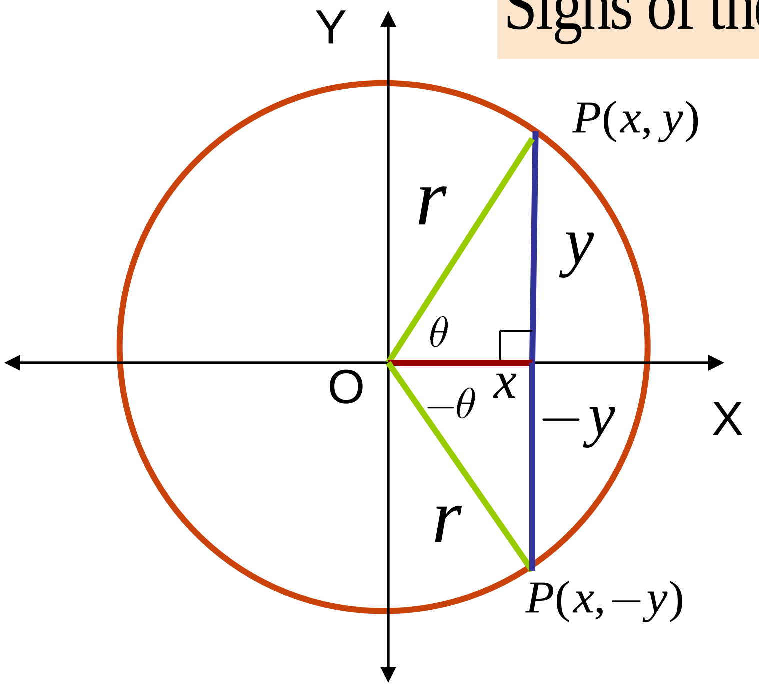
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} = \left(\frac{y}{r}\right) \div \left(\frac{x}{r}\right) = \frac{\sin\theta}{\cos\theta}$$

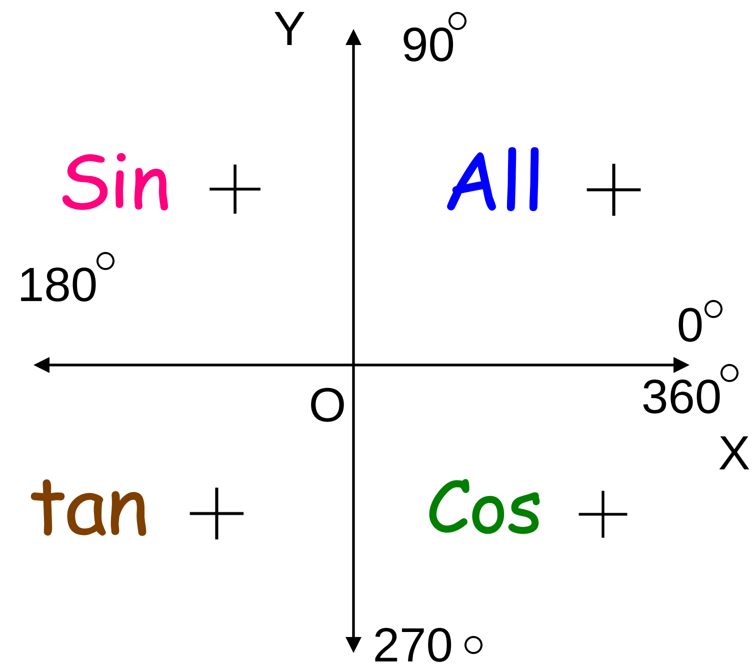
Consider right triangle POB where $\angle POB = \theta$



Representing Angles on the Cartesian Plane



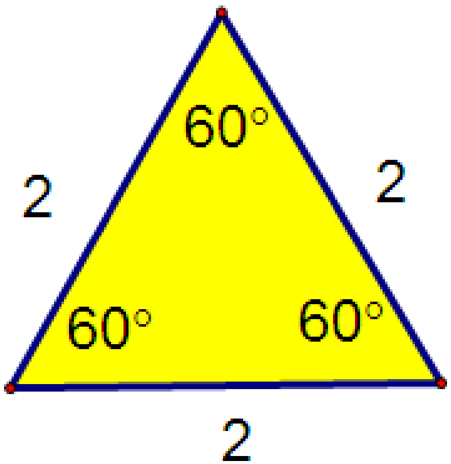
Signs of the trigonometric ratios in the four quadrants

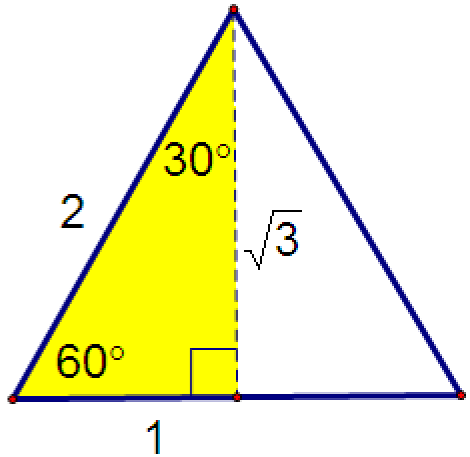


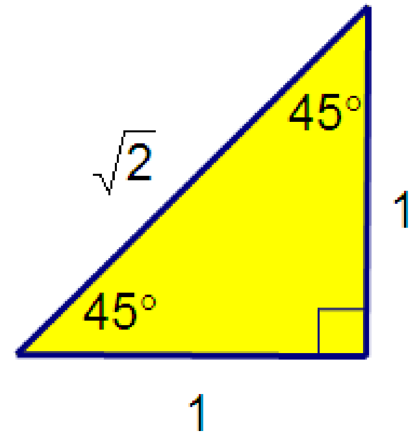
CAST Diagram

1. Signs of co-ordinates x and y changes with quadrants.
2. If OP moves anti-clockwise from pos. X-axis, then θ is positive.
3. If OP moves clockwise from pos. X-axis, then θ is negative.
4. Multiples of $360°$ added to θ give same trigonometric ratios.

Trigonometric Ratios for Standard (special) Angles







$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

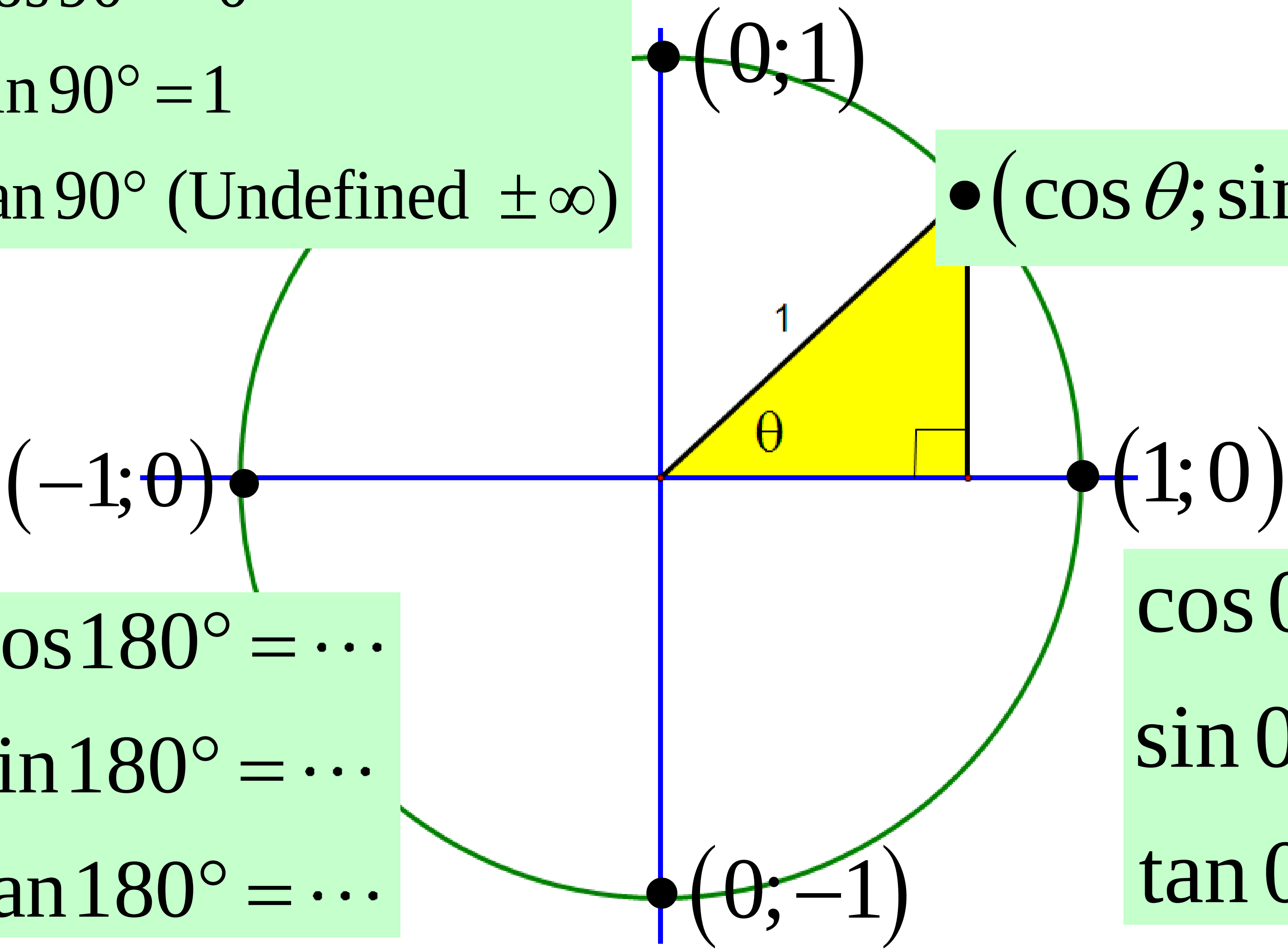
$$\tan 45^\circ = 1$$

Effect of Unit Circle on Trigonometric Ratios

$\cos 90° = 0$

$\sin 90° = 1$

$\tan 90°$ (Undefined $\pm\infty$)



$\cos 180° = \cdots$

$\sin 180° = \cdots$

$\tan 180° = \cdots$

$\cos 0° = 1$

$\sin 0° = 0$

$\tan 0° = 0$

What about sin, cos and tan of 270°?

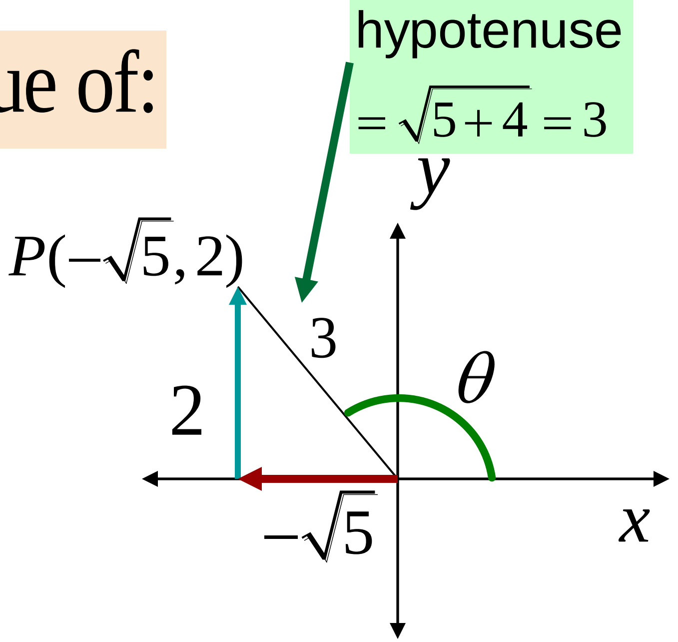
Trigonometric Ratios - Completion of Triangle

Use the diagram and determine the value of:

1. $\cos\theta$ $= -\dfrac{\sqrt{5}}{3}$

2. $\sin(\theta) - 1$ $= \dfrac{2}{3} - 1 = -\dfrac{1}{3}$

3. $2 - \tan^2(\theta)$ $= 2 - \left(\dfrac{2}{-\sqrt{5}}\right)^2 = 2 - \dfrac{4}{5} = 1\dfrac{1}{5}$



Complete the Triangle

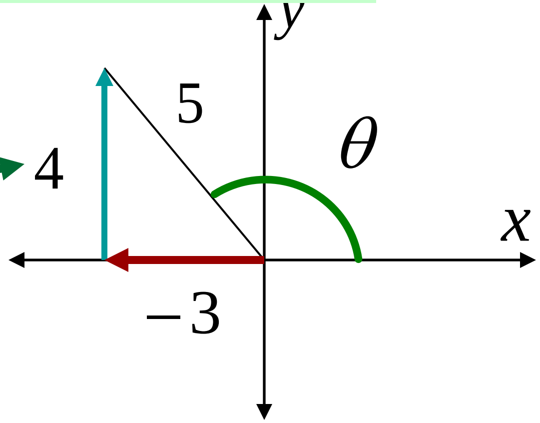
Trigonometric Ratios- Completion of Triangle

Let $\cos(\theta) = -\frac{3}{5}$ and $0° < \theta < 180°$.

Without a calculator, find the value of:

$\cos$ **negative**

$\Rightarrow 2^{nd}$ **quadrant**



Complete the Triangle

$$\frac{\sin(\theta)+\cos(\theta)}{\tan(\theta)}$$

$$\text{opposite} = \sqrt{5^2-(-3)^2} = 4$$

$$= \frac{\frac{4}{5}+\left(\frac{-3}{5}\right)}{-\frac{4}{3}} = \frac{1}{5}\times\left(-\frac{3}{4}\right) = -\frac{3}{20}$$

Tutorial 1 : Standard angles and completion of triangle

Use of calculators are not allowed :

1. Find $2\tan\theta + 13\cos^2\theta$ if $\cos\theta = \frac{-2}{\sqrt{13}}$ and $\theta \in (0°, 180°)$.
2. Use the trigonometric ratios of standard angles to calculate $\frac{\cos 60°}{(\tan^2 45°)} - \sin^2(60°)$

PAUSE DVD

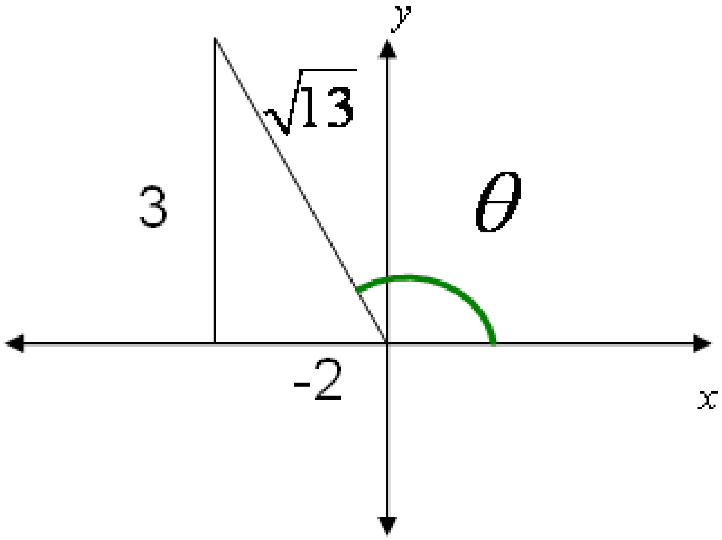
- Do Tutorial 1
- Then View Solutions

Tutorial 1 Problem 1: Completion of Triangle: Suggested Solution

1. $2\tan\theta + 13\cos^2\theta$

$$= 2\times\frac{3}{-2} + 13\times\left(\frac{-2}{\sqrt{13}}\right)^2$$

$$= -3 + 4 = 1$$



Given:

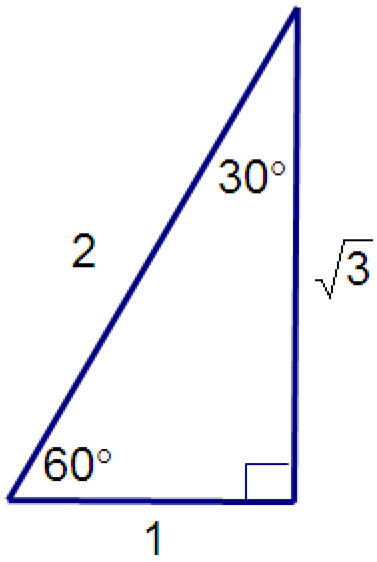
$$\cos\theta = -\frac{2}{\sqrt{13}}$$

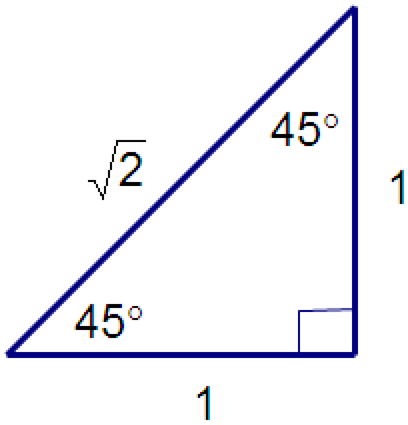
and $\theta \in (0°;180°)$

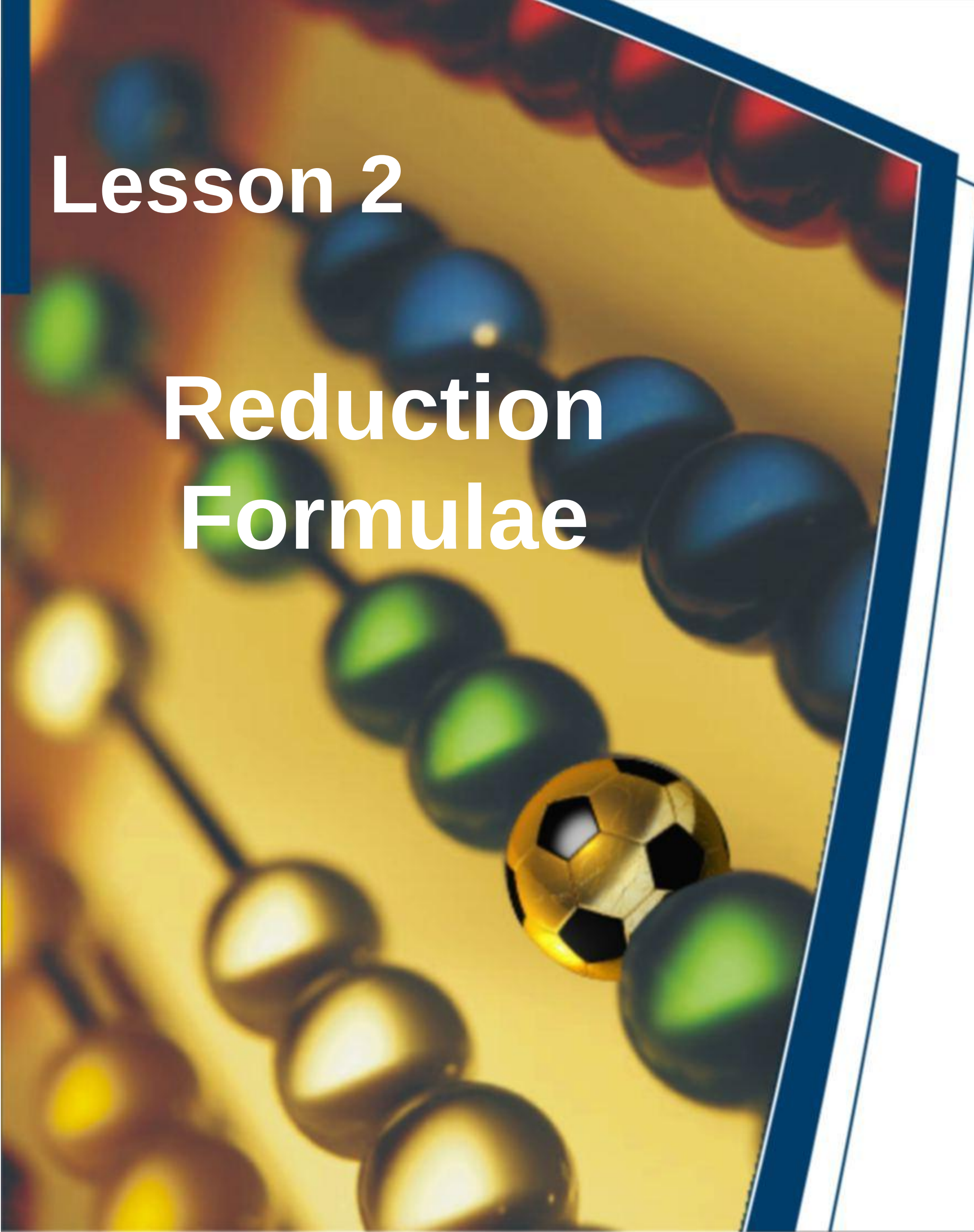
Tutorial 1 Problem 2:
Standard Angles: Suggested Solution

2. $$\frac{\cos 60°}{(\tan^2 45°)} - \sin^2(60°)$$

$$= \frac{1}{2} \div (1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$







Lesson 2

Reduction Formulae

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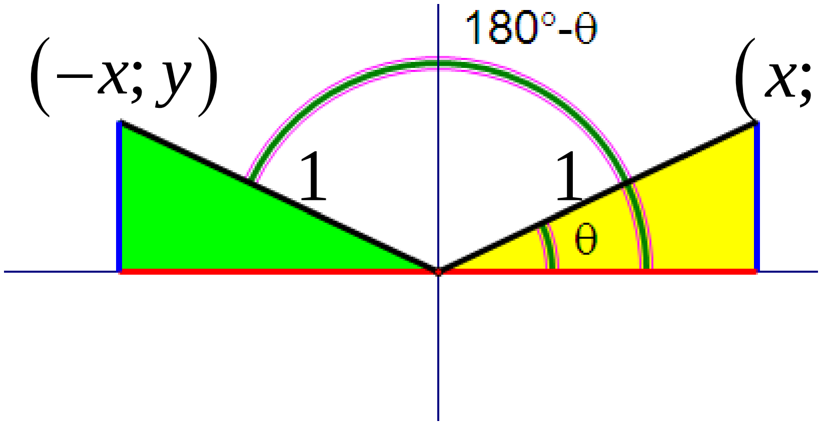
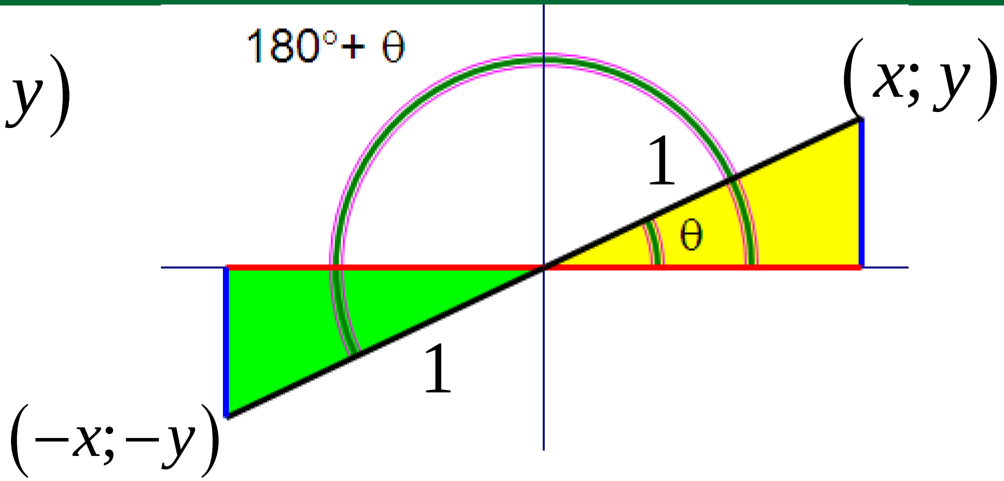
The $(180° + \theta)$ and $(180° - \theta)$ Identities





$$\sin(180° - \theta) = \sin\theta$$
$$\cos(180° - \theta) = -\cos\theta$$
$$\tan(180° - \theta) = -\tan\theta$$

$$\sin(180° + \theta) = -\sin\theta$$
$$\cos(180° + \theta) = -\cos\theta$$
$$\tan(180° + \theta) = \tan\theta$$

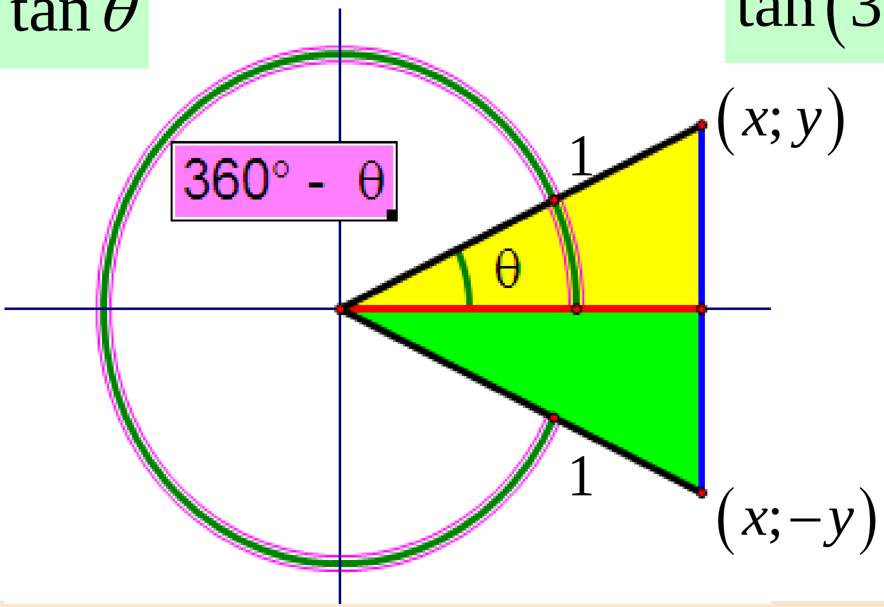
For an angle θ, any trigonometric argument of $(180° + \theta)$ or $(180° - \theta)$ is numerically equal to plus or minus the same trigonometric argument of θ.

The sign may change according to the CAST diagram.

The $(360° - \theta)$ and $(360° + \theta)$ Identities

$$\sin(360° - \theta) = -\sin\theta$$
$$\cos(360° - \theta) = \cos\theta$$
$$\tan(360° - \theta) = -\tan\theta$$

$$\sin(360° + \theta) = \sin\theta$$
$$\cos(360° + \theta) = \cos\theta$$
$$\tan(360° + \theta) = \tan\theta$$



For an angle θ, any trigonometric argument of $(360° - \theta)$ or $(360° + \theta)$ is numerically equal to $\pm$ the same trigonometric argument of θ.

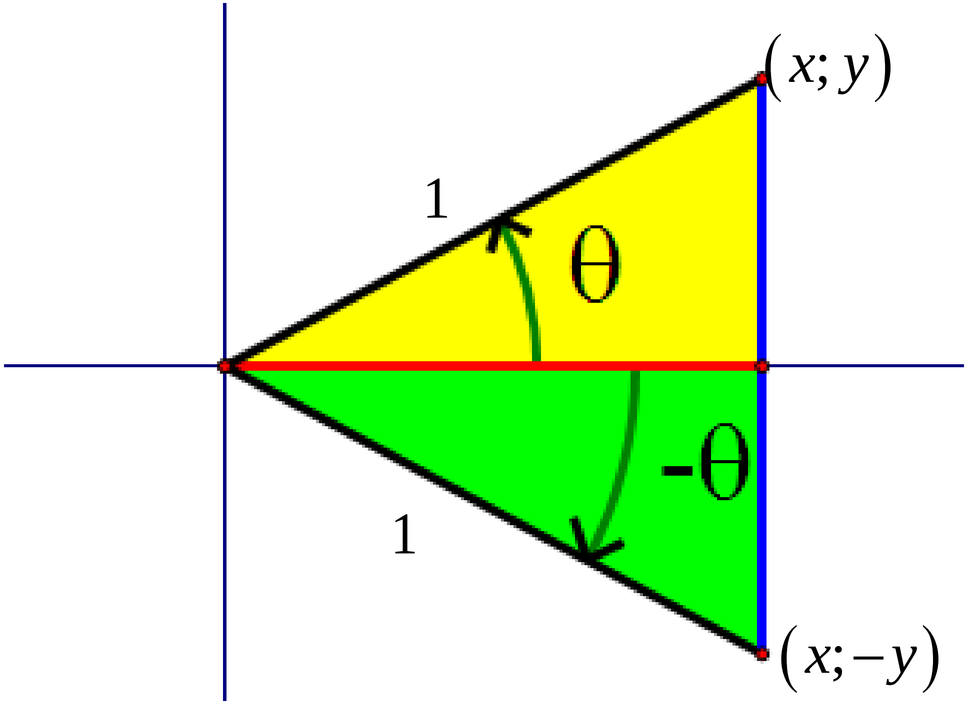
The sign may change according to the CAST diagram.

What about the $(-\theta)$ Identities?

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$



For an angle θ, any trigonometric argument of $(-\theta)$ is numerically equal to $\pm$ the same tirgonometric argument of θ.

The sign may change according to the CAST diagram.

Simplification using Trigonometric Identities

Determine the value of the following without a calculator:

Reference Angle

$\sin(-570°)$

$= -\sin(360° + 210°)$

$= -\sin 210°$

$= -\sin(180° + 30°)$

$= -(-\sin 30°)$

$= \sin 30° = \frac{1}{2}$

$\tan(-135°)$

$= -\tan 135°$

$= -\tan(180° - 45°)$

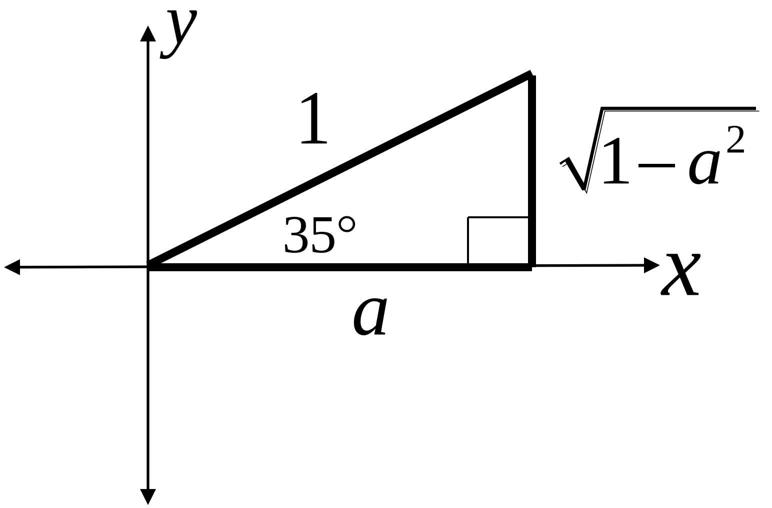
$= -(-\tan 45°)$

$= 1$

More Simplification using Trigonometric Identities

If $\cos(35°) = a$, determine the following i.t.o. a:

Start by drawing the right triangle



1. $\tan(35°)$

$$= \frac{\sqrt{1-a^2}}{a}$$

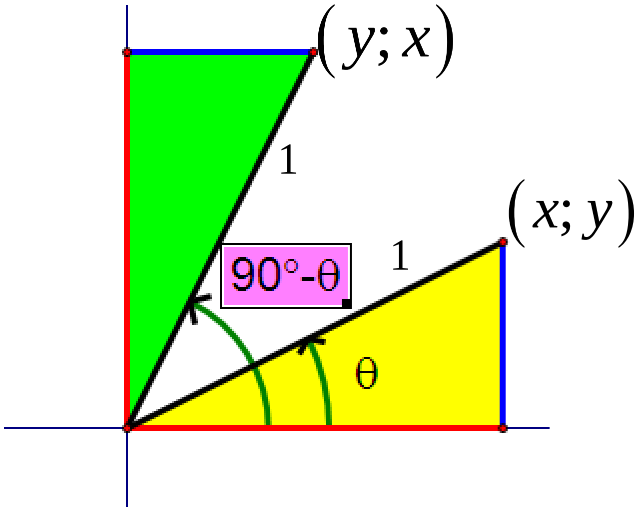
2. $\cos(215°)$

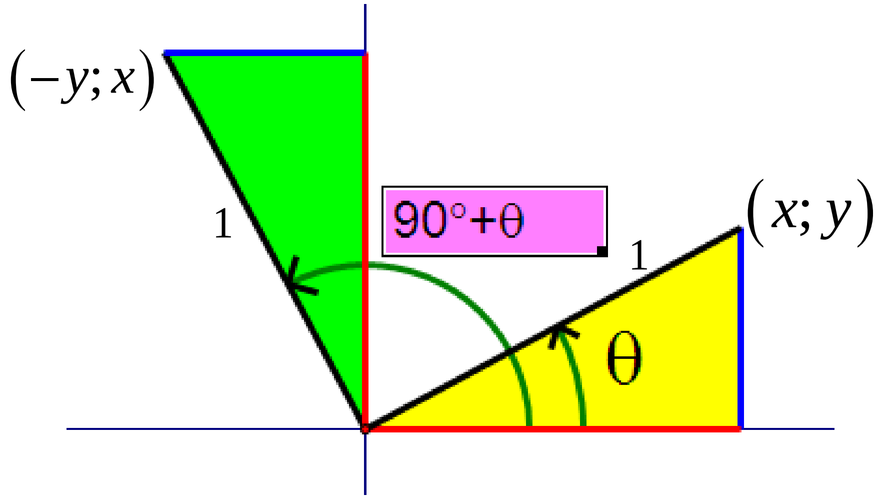
$$= \cos(180° + 35°)$$
$$= -\cos 35°$$
$$= -a$$

Given: $\cos(35°) = \dfrac{a}{1}$

The $(90° \pm \theta)$ Co-function Identities

sin and cos are co-functions of one another





$$\cos(90° - \theta) = \sin\theta$$

$$\sin(90° - \theta) = \cos\theta$$

$$\cos(90° + \theta) = -\sin\theta$$

$$\sin(90° + \theta) = \cos\theta$$

For an angle θ, any trigonometric argument of $(90^\circ \pm \theta)$ is numerically equal to $\pm$ the co-function trigonometric argument of θ.

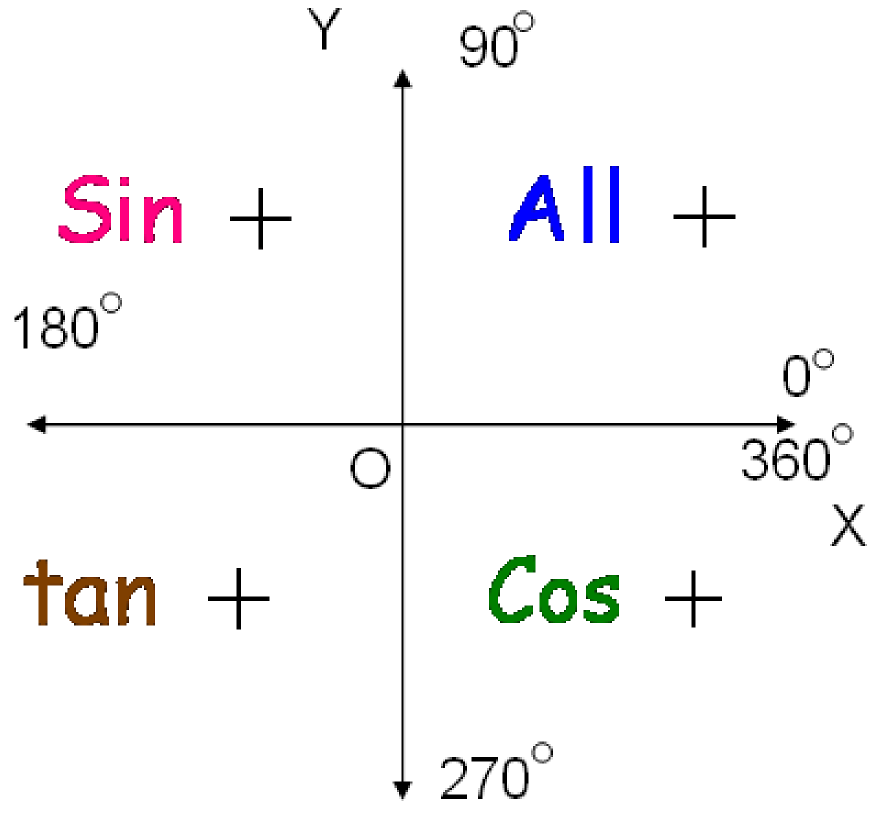
The sign may change according to the CAST diagram.

Applications of the Co-Function Identities

$$\sin 65° = \sin(90° - 25°)$$
$$= \cos 25°$$

$$\cos 118° = \cos(90° + 28°)$$
$$= -\sin 28°$$

$$\sin 146° = \sin(90° + 56°)$$
$$= \cos 56°$$



More examples of Trigonometric Simplification

Calculate without a calculator:

$$\frac{3\cos(150°)\cos(180°)}{\tan(315°)-\cos(240°)}$$

Reference Angles

$$=\frac{3(-\cos 30°)(-1)}{-\tan(45°)-(-\cos 60°)}$$

$$=\frac{(3)\left(-\frac{\sqrt{3}}{2}\right)(-1)}{-1+\frac{1}{2}}=\frac{3\sqrt{3}}{2}\times\left(-\frac{2}{1}\right)=-3\sqrt{3}$$

More examples of Trigonometric Simplification

Simplify the following expressions:

$$\frac{\cos(-\theta)\sin(180° + \theta)}{\tan(\theta - 180°)\sin(\theta + 90°)}$$

$$= \frac{\cos\theta \times (-\sin\theta)}{\tan\theta \times \cos\theta}$$

$$= \frac{\cos\theta \times (-\sin\theta)}{\frac{\sin\theta}{\cos\theta} \times \cos\theta}$$

$$= -\cos\theta$$

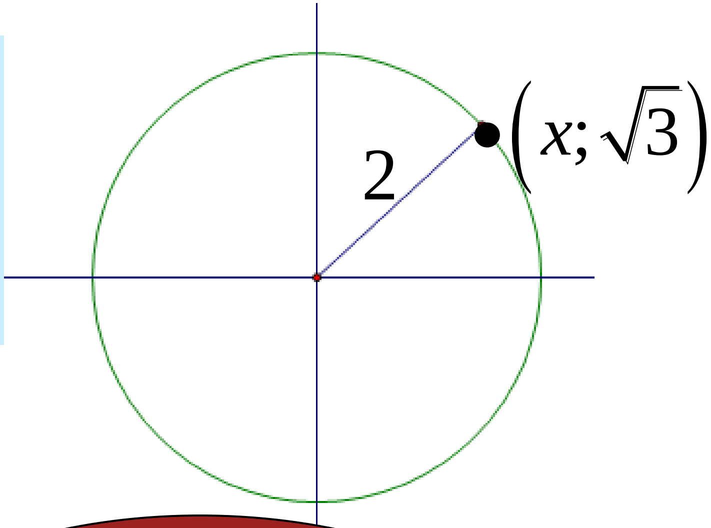
$$\tan(\theta - 180°)$$

$$= \tan\left[-(180° - \theta)\right]$$

$$= -\tan(180° - \theta)$$

$$= \tan\theta$$

Tutorial 2 : Simplification and Reduction



1. Consider the figure and determine:

(a) x (b) $\sin(180° - \theta)$

(c) $\tan(360° - \theta)$ (d) $\cos(90° + \theta)$

2. Calculate without a calculator:

(a) $\sin(570°)$

(b) $\cos(-210°)$

(c) $\tan(315°)$

3. Simplify

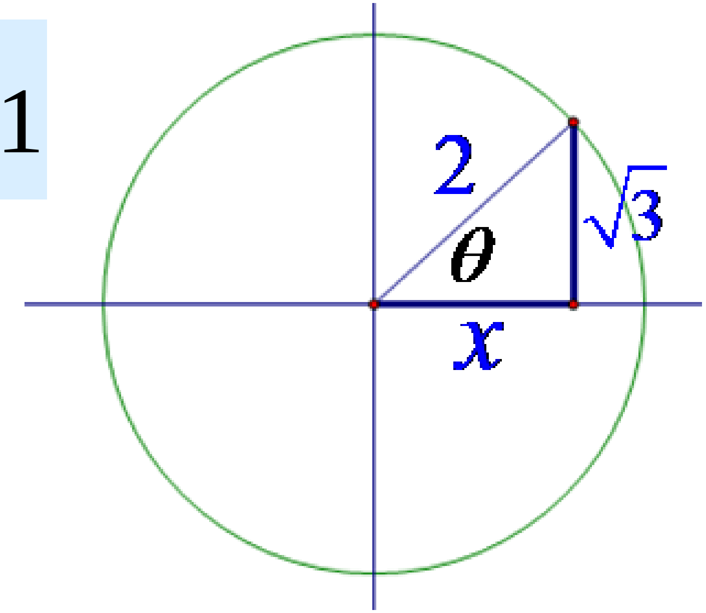
$$\frac{\sin(\theta - 360°)\cos(360° - \theta)}{\tan(180° - \theta)\sin(\theta - 90°)}$$

PAUSE DVD

- Do Tutorial 2
- Then View Solutions

Tutorial 2 Problem 1: Simplification and Reduction: Suggested Solutions

1\. (a) $x = \sqrt{(2)^2 - (\sqrt{3})^2} = 1$



1\. (b) $\sin(180^\circ - \theta)$

$$= \sin\theta = \frac{\sqrt{3}}{2}$$

1\. (c) $\tan(360^\circ - \theta)$

$$= -\tan(\theta) = -\sqrt{3}$$

1\. (d) $\cos(90^\circ + \theta)$

$$= -\sin(\theta) = -\frac{\sqrt{3}}{2}$$

Tutorial 2 Problem 2: Simplification and reduction: Suggested Solutions

2. (a) $\sin(570°) = \sin(360° + 210°) = \sin(210°)$

But $\sin\left(210°\right) = \sin\left(180° + 30°\right)$

$= -\sin 30° = -0,5$

2. (b) $\cos\left(-210°\right)$

$= \cos 210°$

$= \cos\left(180° + 30°\right) = -\cos 30° = -\dfrac{\sqrt{3}}{2}$

2. (c) $\tan\left(315°\right)$

$= \tan\left(360° - 45°\right) = -\tan 45° = -1$

Tutorial 2 Problem 3: Reduction and Simplification: Suggested Solution

3. $$\frac{\sin(\theta - 360^\circ)\cos(360^\circ - \theta)}{\tan(180^\circ - \theta)\sin(\theta - 90^\circ)}$$

$$= \frac{\left(-\sin(360^\circ - \theta)\right)\cos(\theta)}{\left(-\tan\theta\right)\left(-\sin(90^\circ - \theta)\right)}$$

$$= \frac{\left(-(-(\sin(\theta))\right)\cos(\theta)}{\left(-\tan\theta\right)\left(-\cos(\theta)\right)}$$

$$= \frac{\sin\theta \times \cos\theta}{\frac{\sin\theta}{\cos\theta} \times \cos\theta} = \cos\theta$$

Lesson 3

Basic Trigonometric Identities

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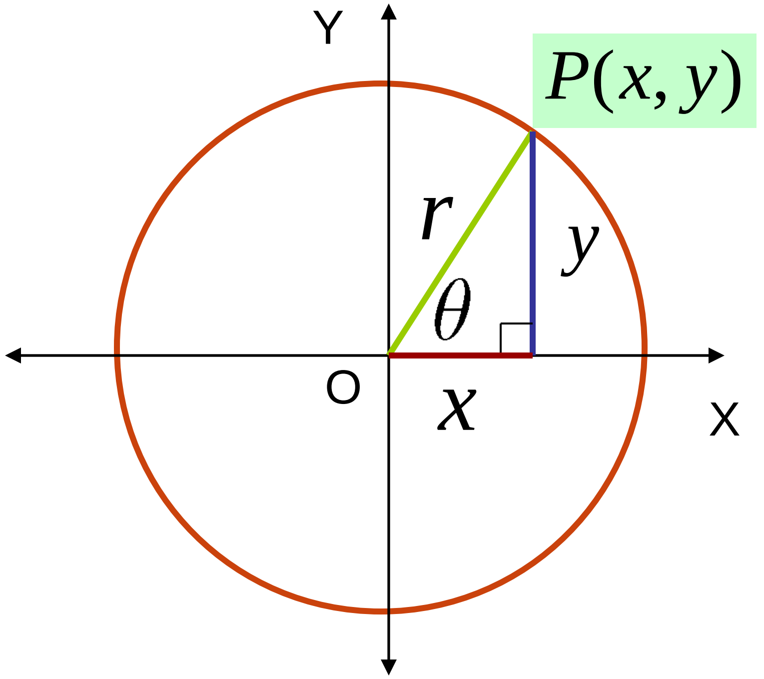
Trigonometric Identities Involving Squares

$$\sin^2\theta + \cos^2\theta$$

$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{y^2 + x^2}{r^2}$$

$$= 1$$



$$\therefore \sin^2\theta + \cos^2\theta = 1$$

Basic Identities: Example1

1. Simplify:

$$\frac{\tan\theta \times \sin\theta}{\cos\theta}$$

Know that: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\dfrac{\sin\theta}{\cos\theta} \times \sin\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta$$

Basic Identities: Example 2

2. Simplify:

$$\frac{\cos\theta \times \sin\theta}{\tan\theta \times \sin(90° + \theta)}$$

$$= \frac{\cos\theta \times \sin\theta}{\frac{\sin\theta}{\cos\theta} \times \cos\theta}$$

$$= \frac{\cos\theta \times \sin\theta}{\sin\theta}$$

$$= \cos\theta$$

Basic Identities: Example 3

3. Simplify: $\sin^2\theta \times \left[\dfrac{1}{\cos^2\theta - 1} \right]$

$$= \sin^2\theta \times \left[\frac{1}{-\sin^2\theta} \right]$$

$$= -1$$

Know that: $\sin^2\theta + \cos^2\theta = 1$

Hence: $\cos^2\theta - 1 = -\sin^2\theta$

Tutorial 3 : Simple identities

Simplify:

PAUSE DVD

- Do Tutorial 3
- Then View Solutions

1. $$\frac{\tan\theta \times \cos(-\theta)}{\sin(90° - \theta)}$$

2. $$\frac{1}{\cos^2\theta}\left[(1+\sin\theta)(1-\sin\theta)\right]$$

Tutorial 3 Problem 1: Basic Identities: Suggested Solution

1. $$\frac{\tan\theta \times \cos(-\theta)}{\sin(90° - \theta)}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} \times \cos\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Tutorial 3 Problem 2: Simple identities: Suggested Solution

2. $$\frac{1}{\cos^2\theta}\left[(1+\sin\theta)(1-\sin\theta)\right]$$

$$=\frac{1}{\cos^2\theta}\times\left(1-\sin^2\theta\right)$$

$$=\frac{1}{\cos^2\theta}\times\cos^2\theta$$

$$=1$$

<u>Know:</u> $\sin^2\theta+\cos^2\theta=1$

End of the DVD on
Basic Trigonometric Relationships

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in the DVD.
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!