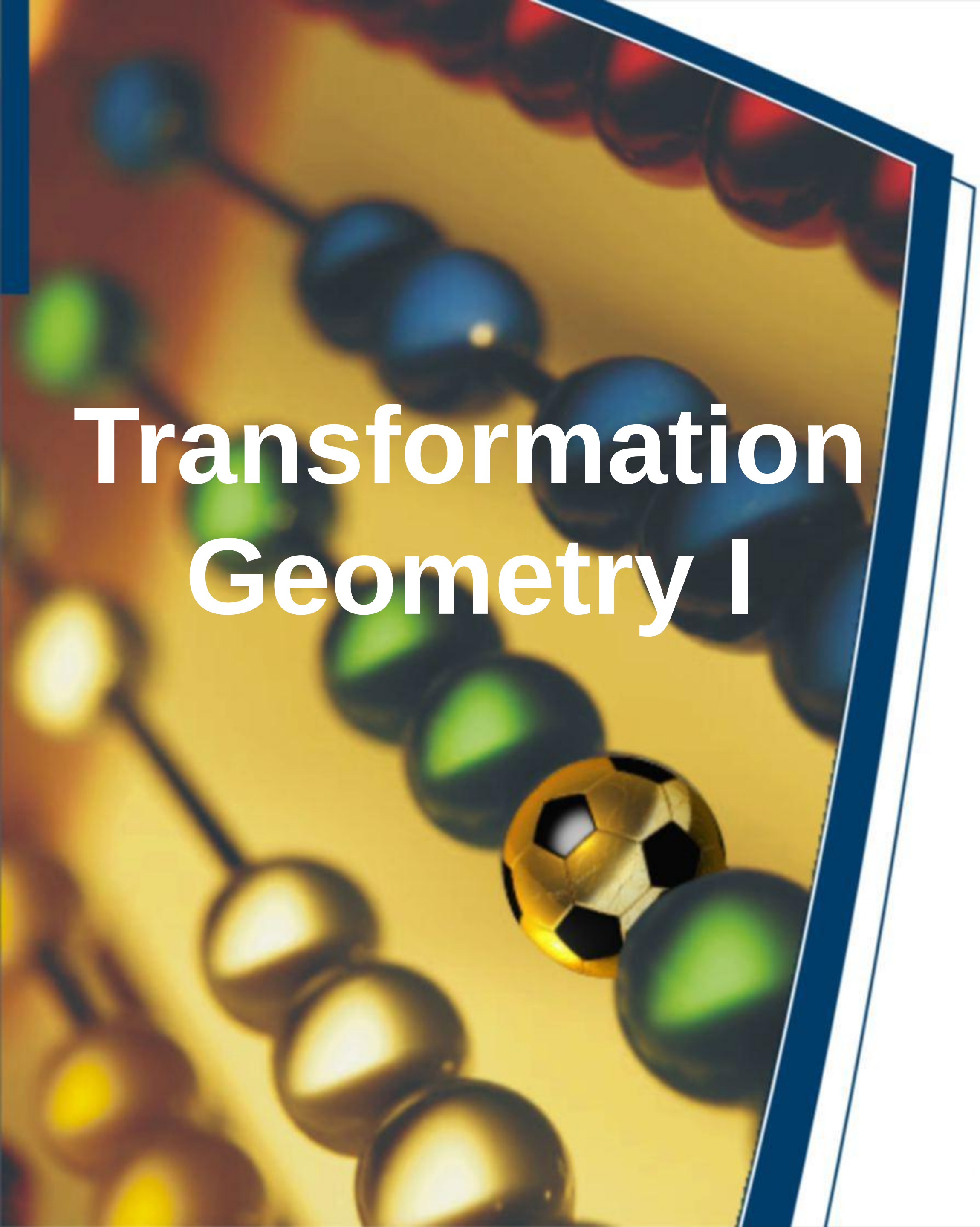

Transformation Geometry I

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NCS Mathematics DVD Series

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Outcomes for this DVD

In this DVD we will:

- Revise translation and reflection transformations of the point (*x; y*). LESSON 1
- Investigate rotation of the point *(x; y)* around the origin through an angle of 90º and 180º in a clockwise and anti-clockwise direction. LESSON 2
- Investigate the vertices of a polygon after enlargement through the origin by a constant factor k. LESSON 3

Lesson 1

Review of translation and reflection of points

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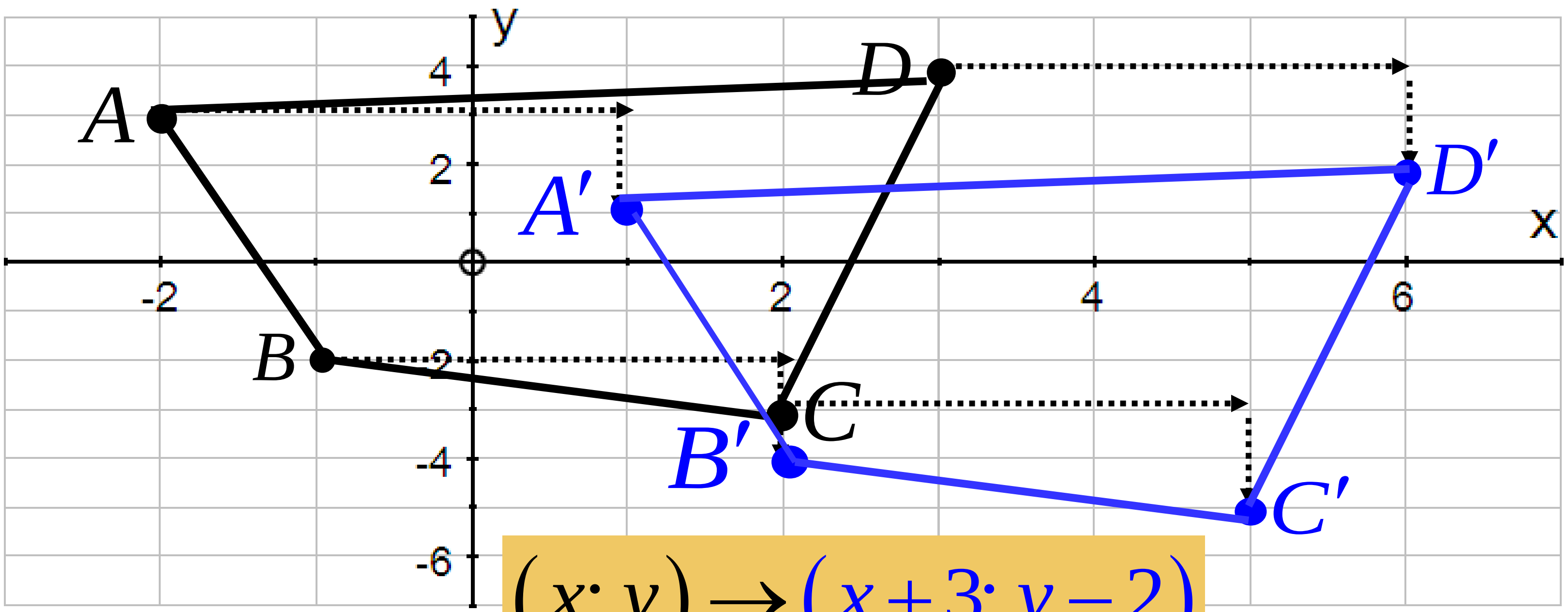
Revision: Translation of points

Draw on graph paper the quadrilateral with the following vertices: $A(-2;3)$; $B(-1;-2)$; $C(2;-3)$ and $D(3;4)$.



$$(x;y)\rightarrow(x+3;y-2)$$

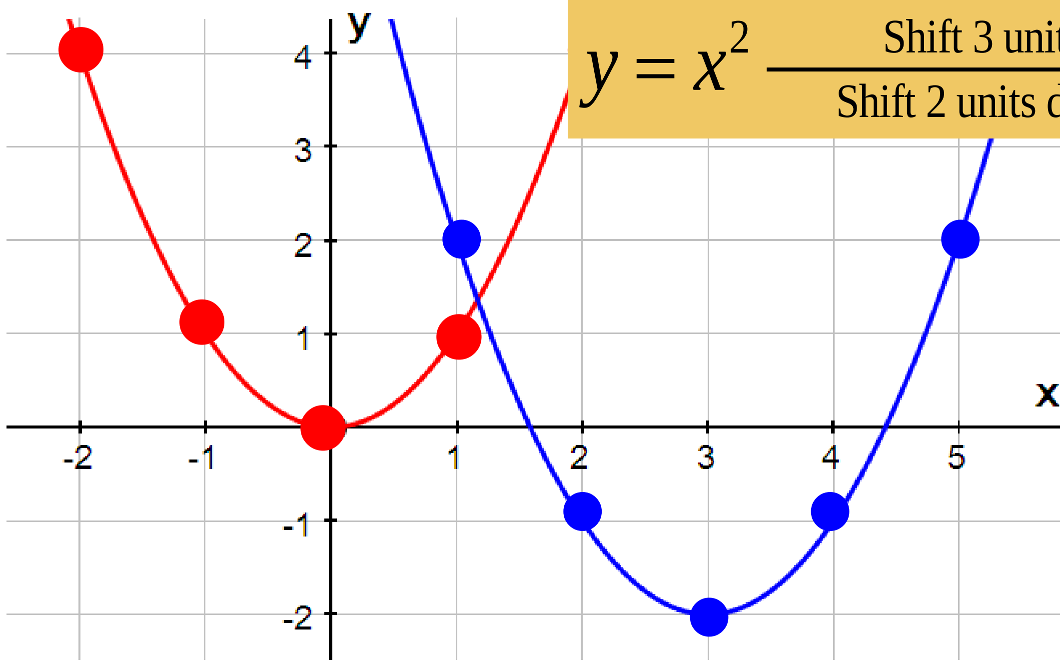
Shift vertices of quadrilateral $ABCD$ 3 units to the right and 2 units downwards. Indicate the coordinates of the vertices and draw the transformation $A'B'C'D'$.

Revision: Translation of a Relation

All points on parabola defined by $y = x^2$ is transformed by the rule $(x; y) \xrightarrow[\text{Shift 2 units downward}]{\text{Shift 3 units to right}} (x+3; y-2)$.

- Sketch the original and transformed parabolas.
- Write down the defining equation for the transformed parabola.

$$y = x^2 \xrightarrow[\text{Shift 2 units downward}]{\text{Shift 3 units right}} y = (x-3)^2 - 2$$

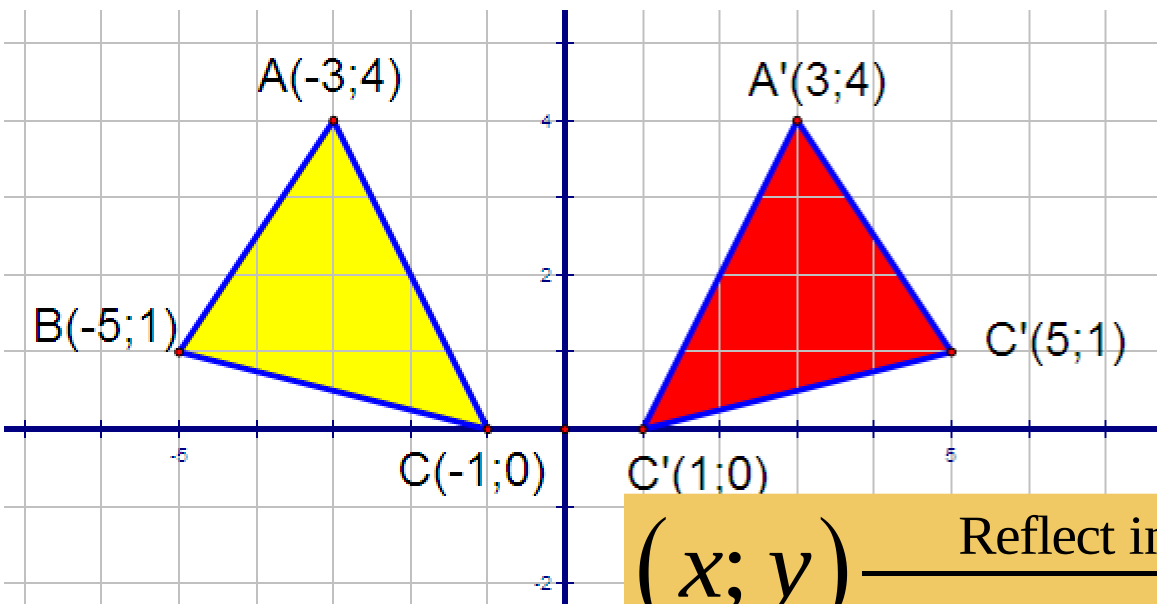


Revision: Reflection in y - axis

ΔABC with vertices $A(-3;4)$; $B(-5;1)$ and $C\ (-1;0)$ is reflected about the $y-$axis to form $\Delta A'B'C'$.

(a) Indicate how to obtain the transformed triangle.

(b) Generalize this type of transformation.

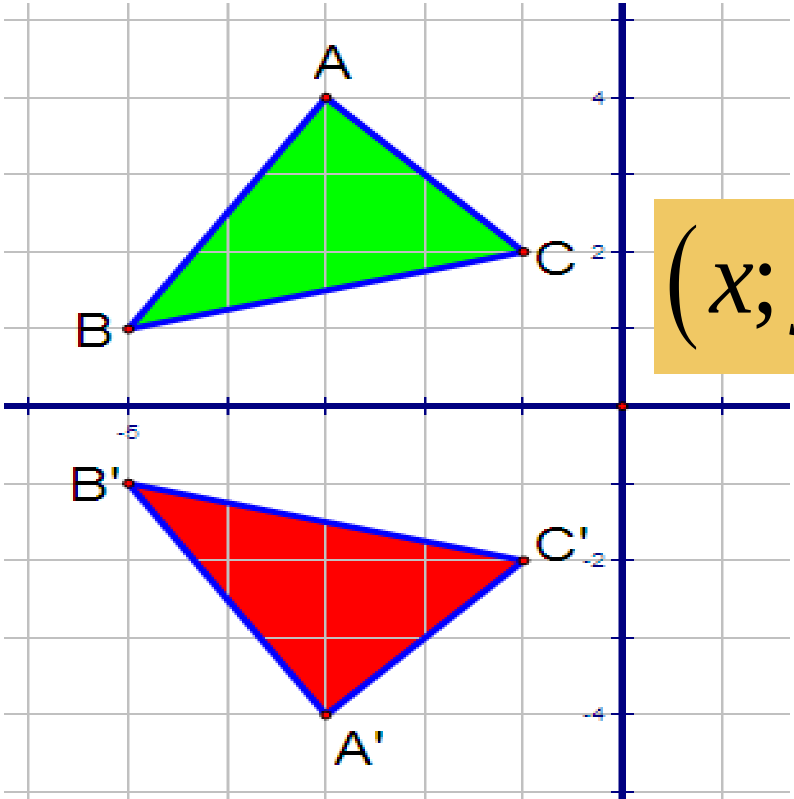


$$(x;y) \xrightarrow{\text{Reflect in } y\text{–axis}} (-x;y)$$

Revision: Reflection in x - axis

ΔABC with vertices $A(-3;4)$; $B(-5;1)$ and $C\ (-1;2)$ is reflected about the $x-$axis to form $\Delta A'B'C'$.

- Indicate how to obtain the transformed triangle.
- Generalize this type of transformation.

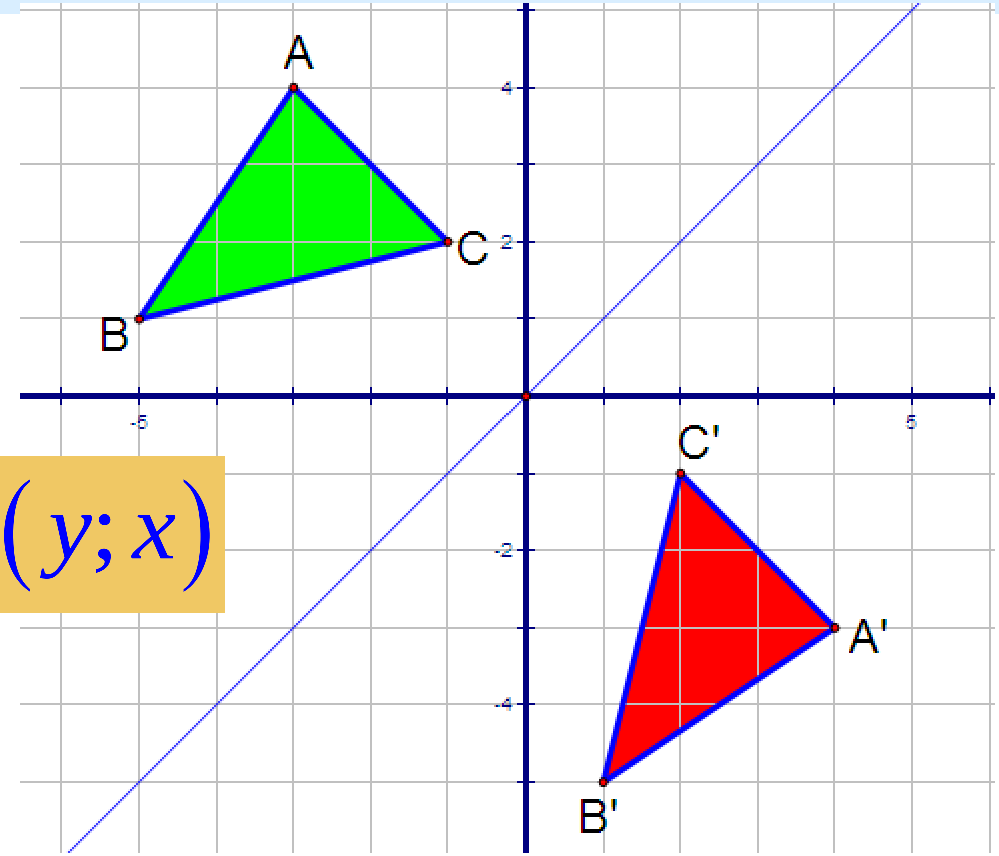


$$(x;y) \xrightarrow{\text{Reflect in } x-\text{axis}} (x;-y)$$

Revision: Reflection in line y = x

ΔABC is reflected in the line $y = x$.

- Determine transformed $\Delta A'B'C'$
- Generalize this type of transformation.



$$(x; y) \xrightarrow{\text{Reflect in } y=x} (y; x)$$

Revision: Reflection of a Relation

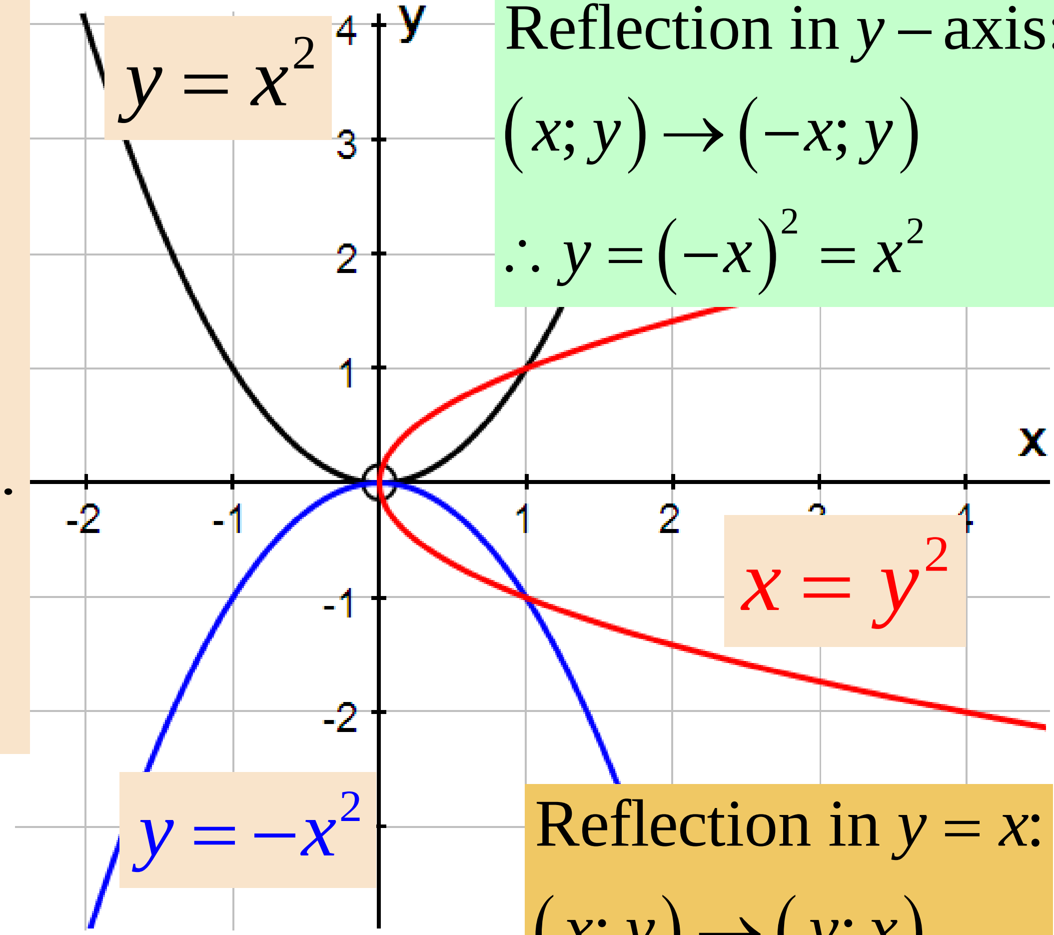
Relation defined by $y = x^2$ is

(a) Reflected in $y-$axis

(b) Reflected in $x-$axis

(c) Reflected in line $y = x$

- Sketch the original and the three transformed relations.
- Write down the defining equations for each transformed relation.



Reflection in $y-$axis:

$(x; y) \to (-x; y)$

$\therefore y = (-x)^2 = x^2$

Reflection in $x-$axis:

$(x; y) \to (x; -y)$

$\therefore -y = x^2 \Rightarrow y = -x^2$

Reflection in $y = x$:

$(x; y) \to (y; x)$

$\therefore x = y^2$

Tutorial 1: Part 1: Translation

1. $A(-2;4)$; $B(3;1)$ and $C(0;-3)$ are the vertices of ΔABC.
   ΔABC is shifted 2 units left and 3 units downwards to form ΔDEF.
   (a) Sketch and label the original and transformed triangles.
   (b) Generalize this transformation.

PAUSE DVD

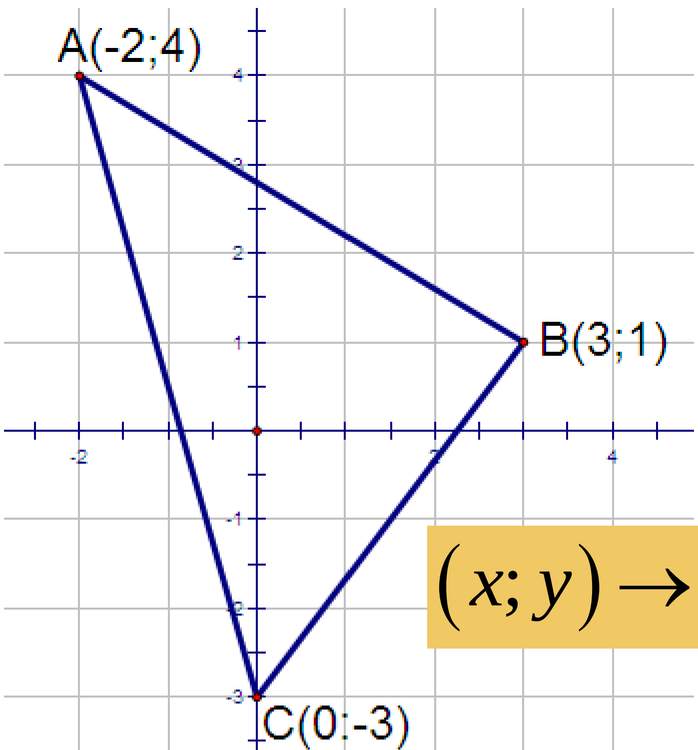
- Do Tutorial 1 Part 1
- Then View Solutions

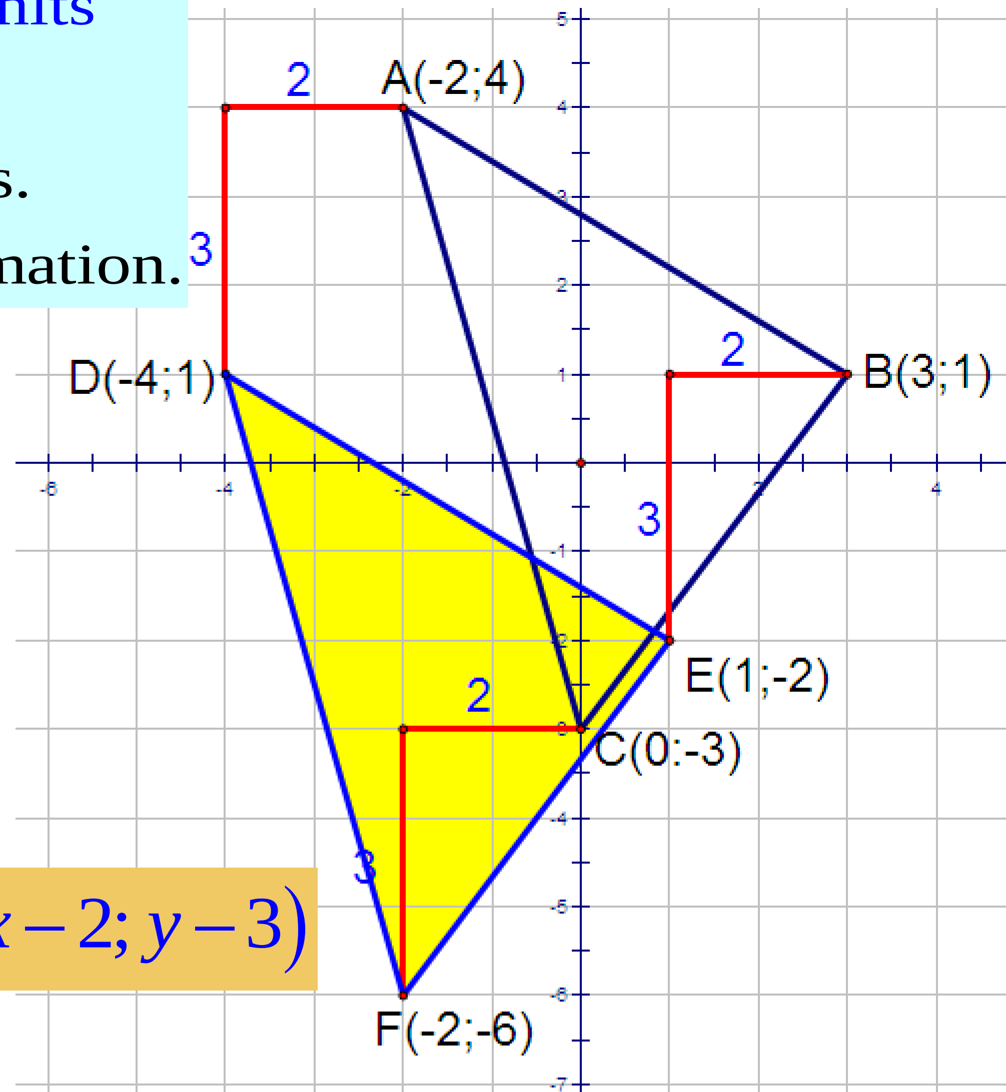
Tutorial 1 Problem 1: Suggested Solution

1. $A(-2;4)$; $B(3;1)$ and $C(0;-3)$ is shifted 2 units left and 3 units downwards to form ΔDEF.

   (a) Sketch and label both Δ's.

   (b) Generalize this transformation.





$$(x;y) \rightarrow (x-2;y-3)$$

Tutorial 1: Part 2: Reflection

2. Given the sketch of ΔABC with vertices $A(-2;4)$; $B(3;1)$ and $C(0;-3)$.

(a) Draw and label $\Delta A'B'C'$ which is the reflection of ΔABC in the $x-$axis.

(b) ΔABC is reflected in the line $y = x$ to form $\Delta A''B''C''$. Sketch and label $\Delta A''B''C''$.

(c) Generalize these two transformations.

PAUSE DVD

- Do Tutorial 1 Part 2
- Then View Solutions

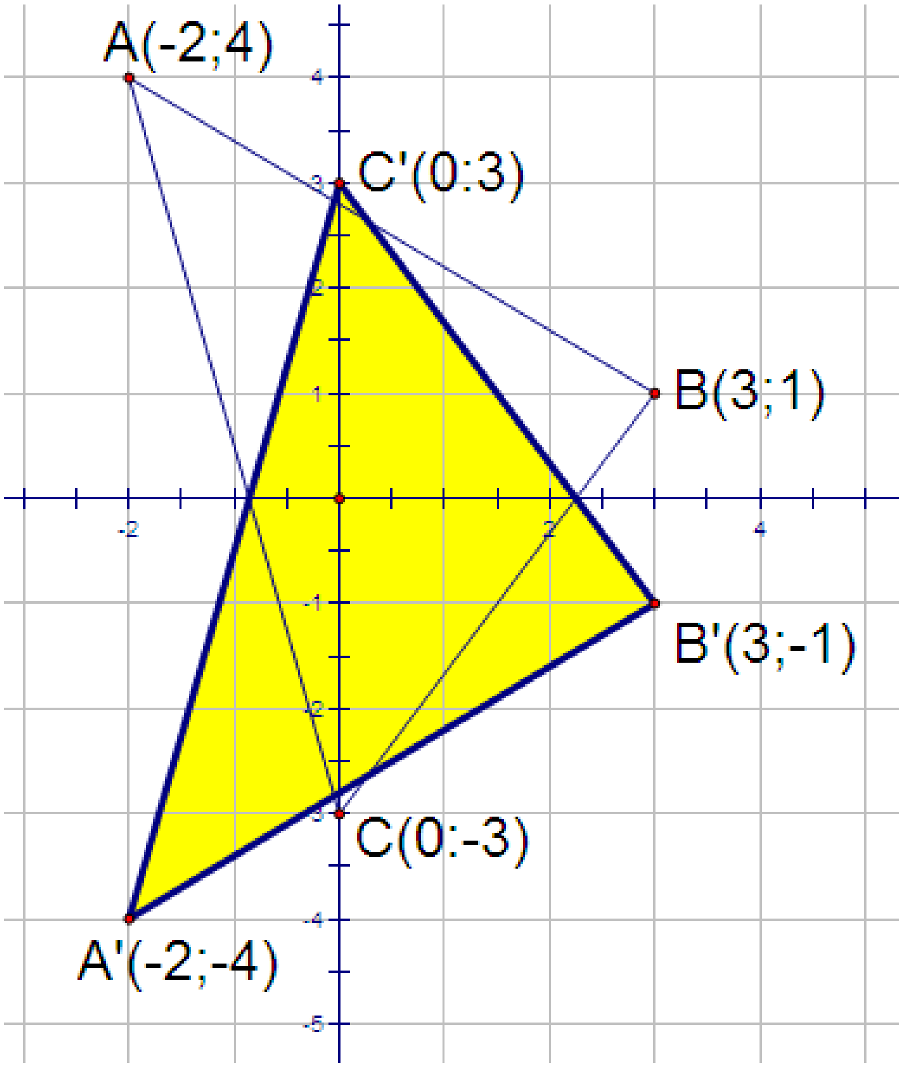
Tutorial 1 Problem 2 (a) & (c): Suggested Solutions

2. $A(-2;4)$; $B(3;1)$ and $C(0;-3)$.

   (a) Draw and label $\Delta A'B'C'$ which is the reflection of ΔABC in the $x-$axis.

   (c) Generalize this transformation.

$$(x;y) \rightarrow (x;-y)$$



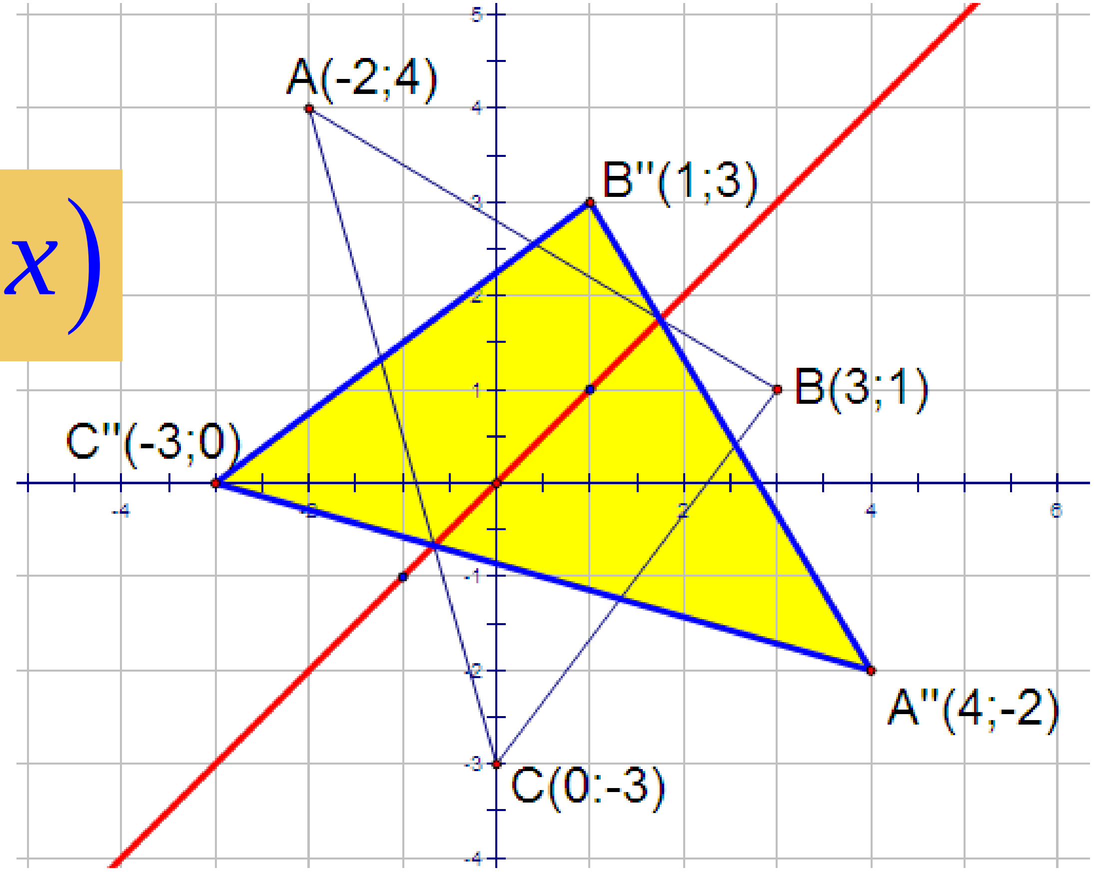
Tutorial Problem 2 (b) & (c): Suggested Solutions

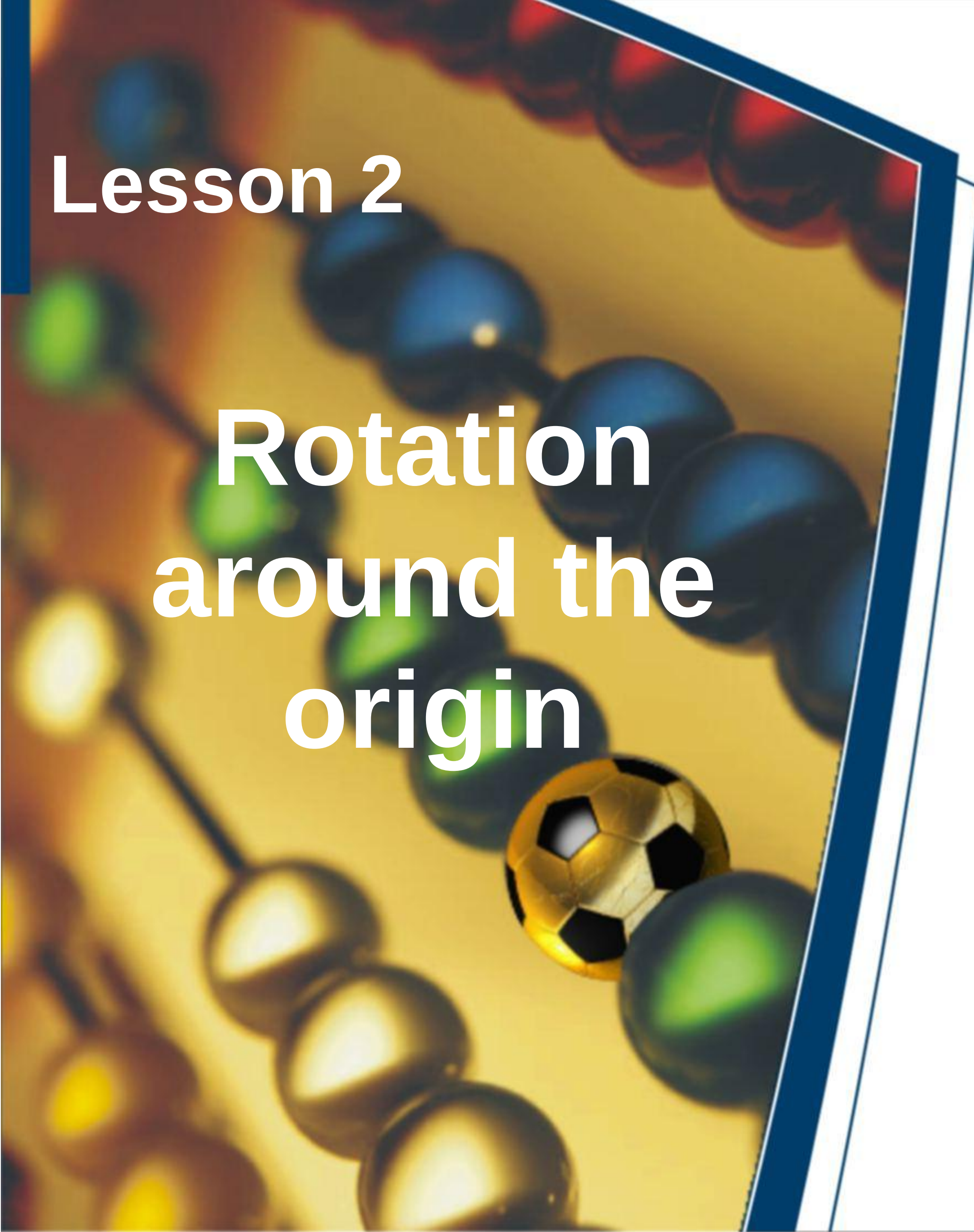
2. (b) ΔABC is reflected in the line $y = x$ to form $\Delta A''B''C''$.

   Sketch and label $\Delta A''B''C''$.

   (c) Generalize this transformation.

$$\left(x; y\right) \rightarrow \left(y; x\right)$$





Lesson 2

Rotation around the origin

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Rotation of points around the origin

We will investigate the effect on co-ordinates of a point $(x; y)$ after rotation around the origin through angles of:

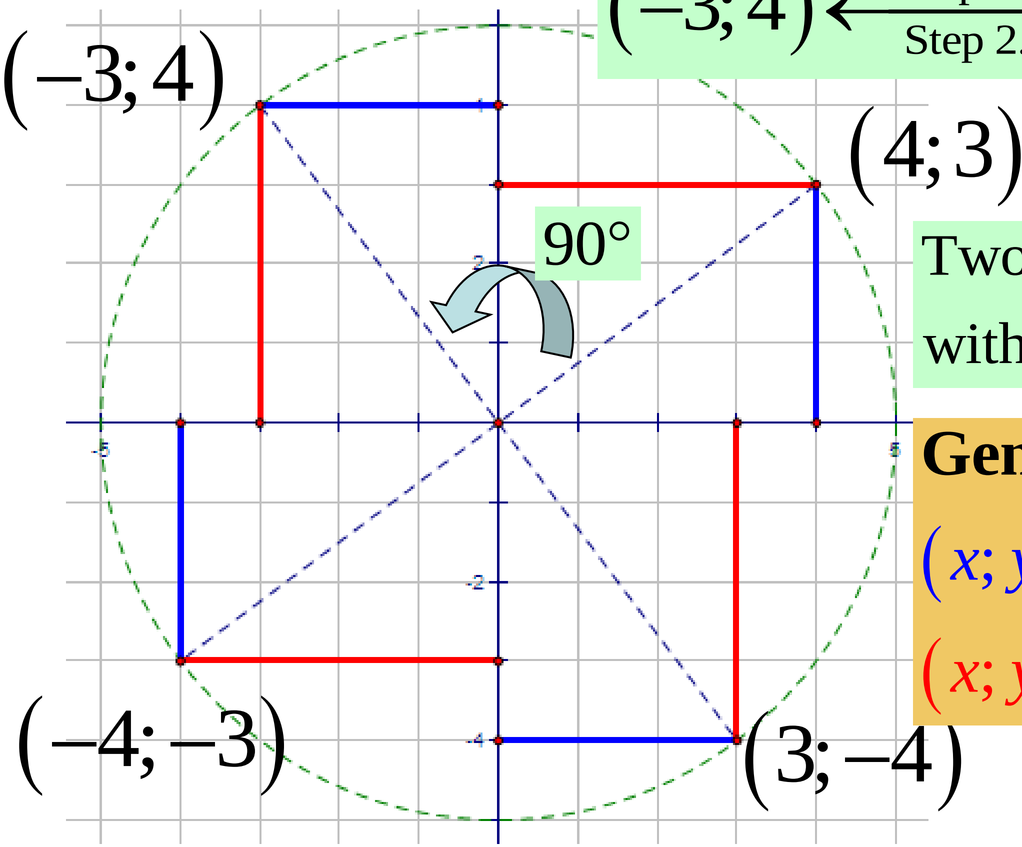
1) $90°$ (Anti-clockwise)
2) $-90°$ (Clock-wise)
3) $\pm\ 180°$

Then we will generalize these effects.

Rotation of a point through 90° or 180° in an anti-clockwise direction around the origin.

Investigate rotation of the point $(4;3)$ continuously through 90° in an anti-clockwise direction around the origin.

$$(-3;4) \xleftarrow[\text{Step 2. Interchange components}]{\text{Step 1. Change sign of } y-\text{component}} (4;3)$$



Two steps are repeated as we continue with rotation process **(CHECK!).**

Generalization :

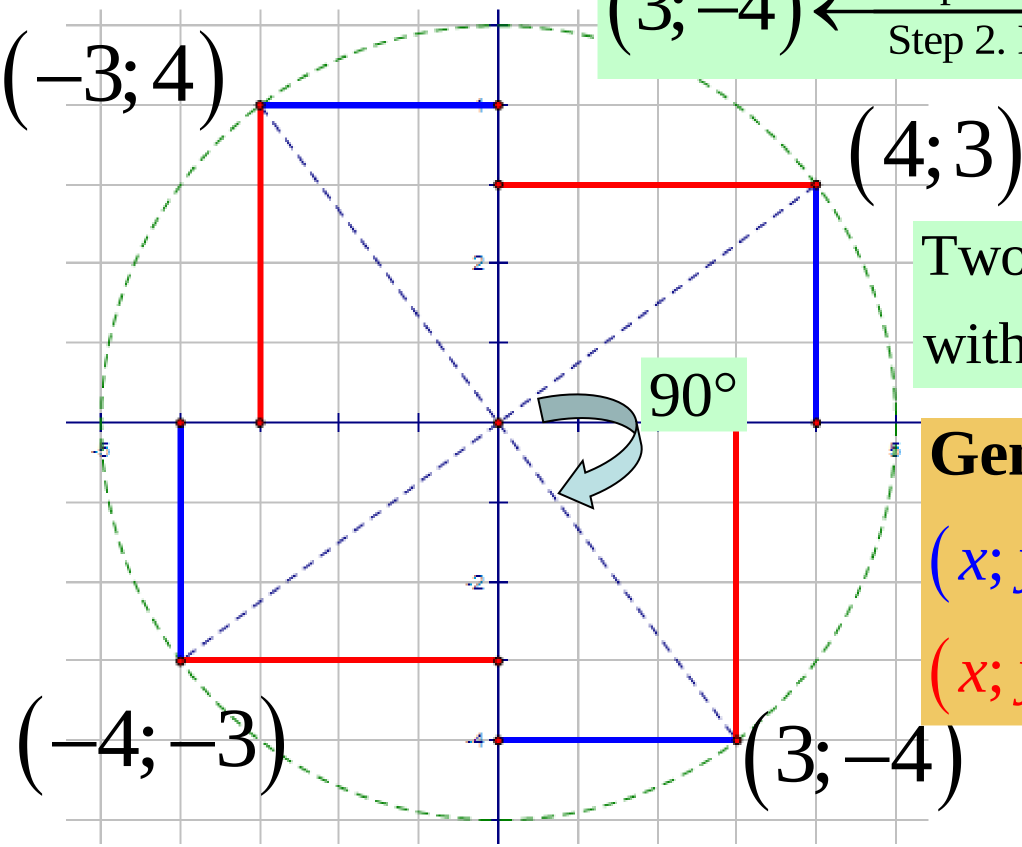
$$(x;y) \xrightarrow[\text{anti-clockwise direction}]{\text{Rotate through } 90^\circ \text{ in an}} (-y;x)$$

$$(x;y) \xrightarrow[\text{anti-clockwise direction}]{\text{Rotate through } 180^\circ \text{ in an}} (-x;-y)$$

Rotation of a point through 90° or 180° in a clockwise direction around the origin.

Investigate rotation of the point $(4;3)$ continuously through 90° in a clockwise direction around the origin.

$$(3;-4) \xleftarrow[\text{Step 2. Interchange components}]{\text{Step 1. Change sign of } x-\text{component}} (4;3)$$



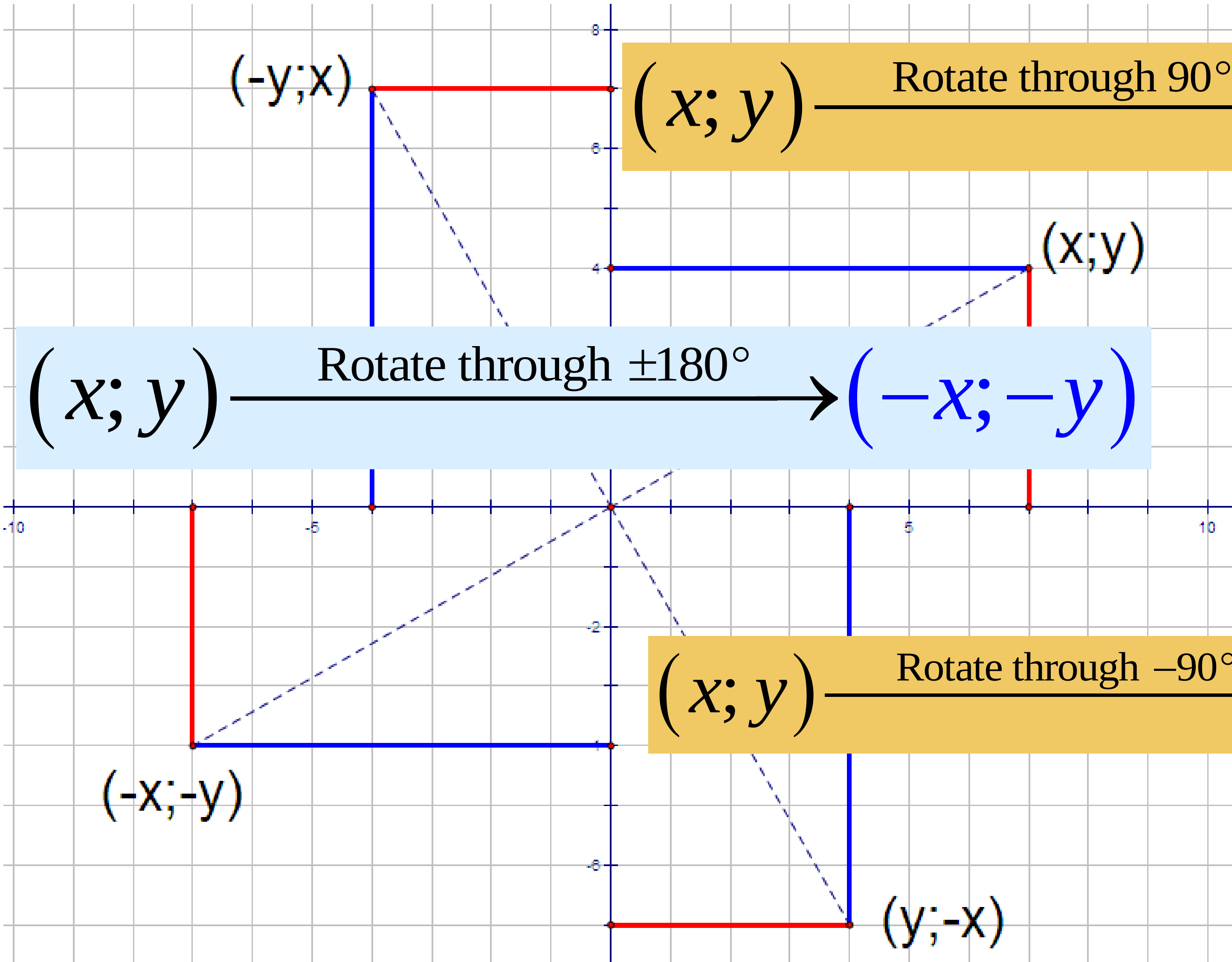
Two steps are repeated as we continue with rotation process **(CHECK!).**

Generalization :

$$(x;y) \xrightarrow[\text{clockwise direction}]{\text{Rotate through } 90^\circ \text{ in a}} (y;-x)$$

$$(x;y) \xrightarrow[\text{clockwise direction}]{\text{Rotate through } 180^\circ \text{ in a}} (-x;-y)$$

Generalize the effect on co-ordinates of a point after rotation around the origin through $\pm 90°$ or $\pm 180°$.



$$(x; y) \xrightarrow{\text{Rotate through } 90°} (-y; x)$$

$$(x; y) \xrightarrow{\text{Rotate through } \pm 180°} (-x; -y)$$

$$(x; y) \xrightarrow{\text{Rotate through } -90°} (y; -x)$$

Rotation of a point through angles of $90°;\ -90°$ and $\pm 180°$ about the origin.

The point $(-3;-4)$ is rotated through angles of $90°;\ -90°;\ 180°$ and $-180°$ about the origin.

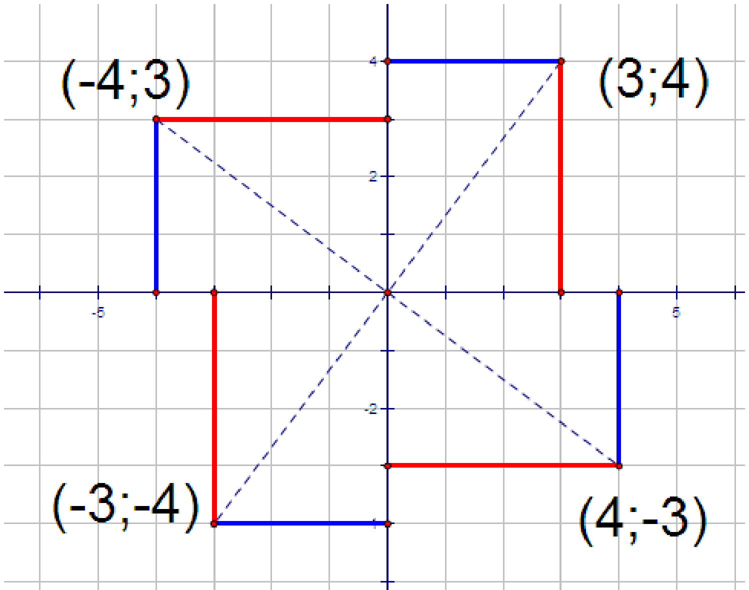
Write down the co-ordinates of the transformed points under each of the four transformations.

$$(-3;-4)\xrightarrow{90°}(4;-3)$$

$$(-3;-4)\xrightarrow{-90°}(-4;3)$$

$$(-3;-4)\xrightarrow{180°}(3;4)$$

$$(-3;-4)\xrightarrow{-180°}(3;4)$$



Tutorial 2: Part 1: Rotation about the origin

1. Write down the co-ordinates of the transformed points under the following rotations about the origin:

(a) $(-2;5)$ is rotated in an anti-clockwise direction through $90°$.

(b) $(3;-4)$ is rotated in a clockwise direction through $90°$.

(c) $(-3;7)$ is rotated through $\pm 180°$.

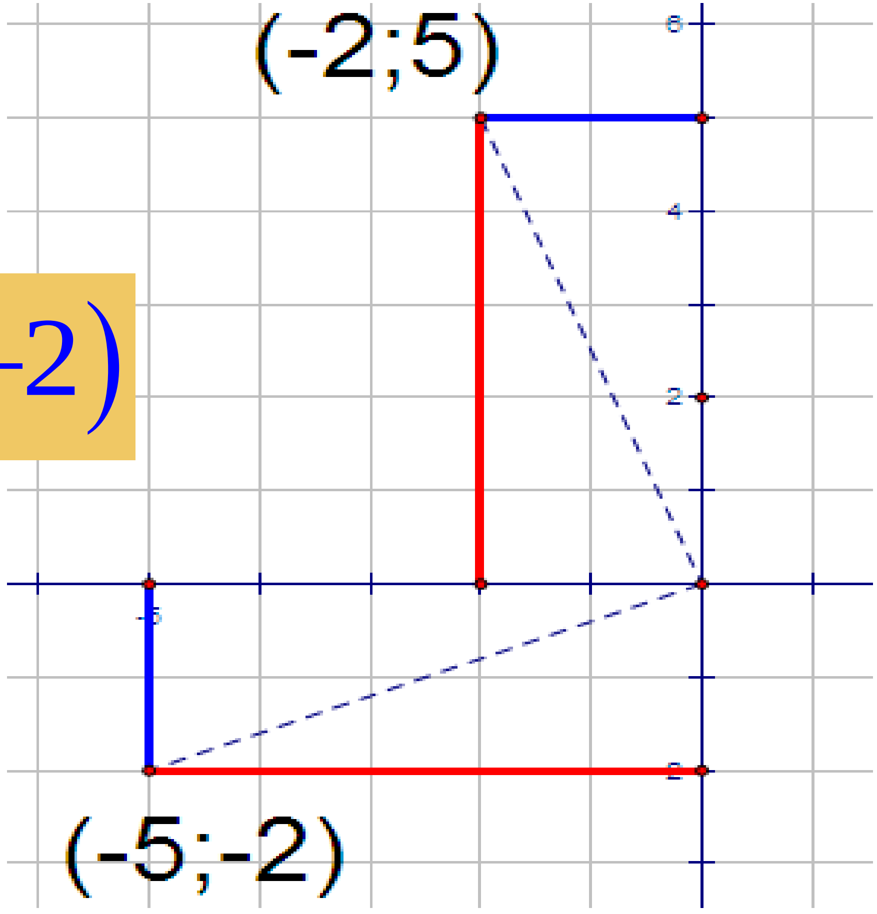
PAUSE DVD

- Do Tutorial 2 Part 1
- Then View Solutions

Tutorial 2 Problem 1(a): Suggested Solution

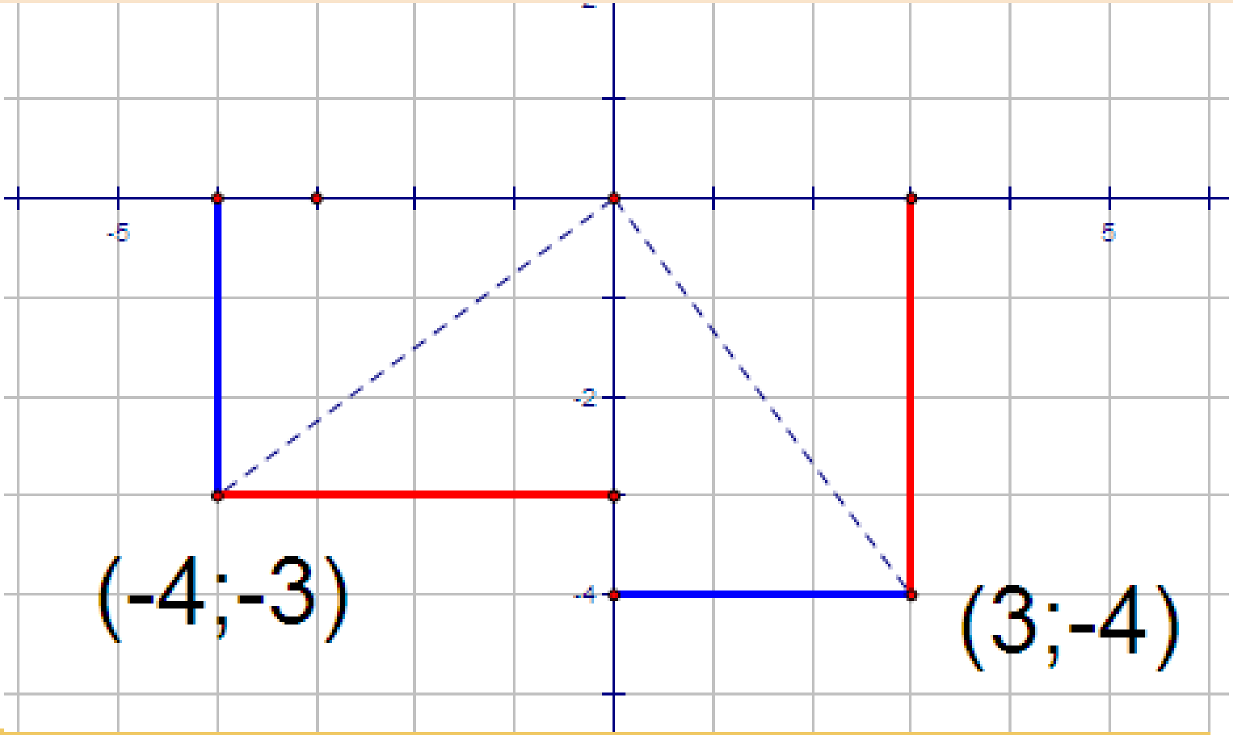
(a) $(-2;5)$ is rotated in an anti - clockwise direction through $90°$.

$$(-2;5) \xrightarrow{90°} (-5;-2)$$



Tutorial 2 Problem 1(b): Suggested Solution

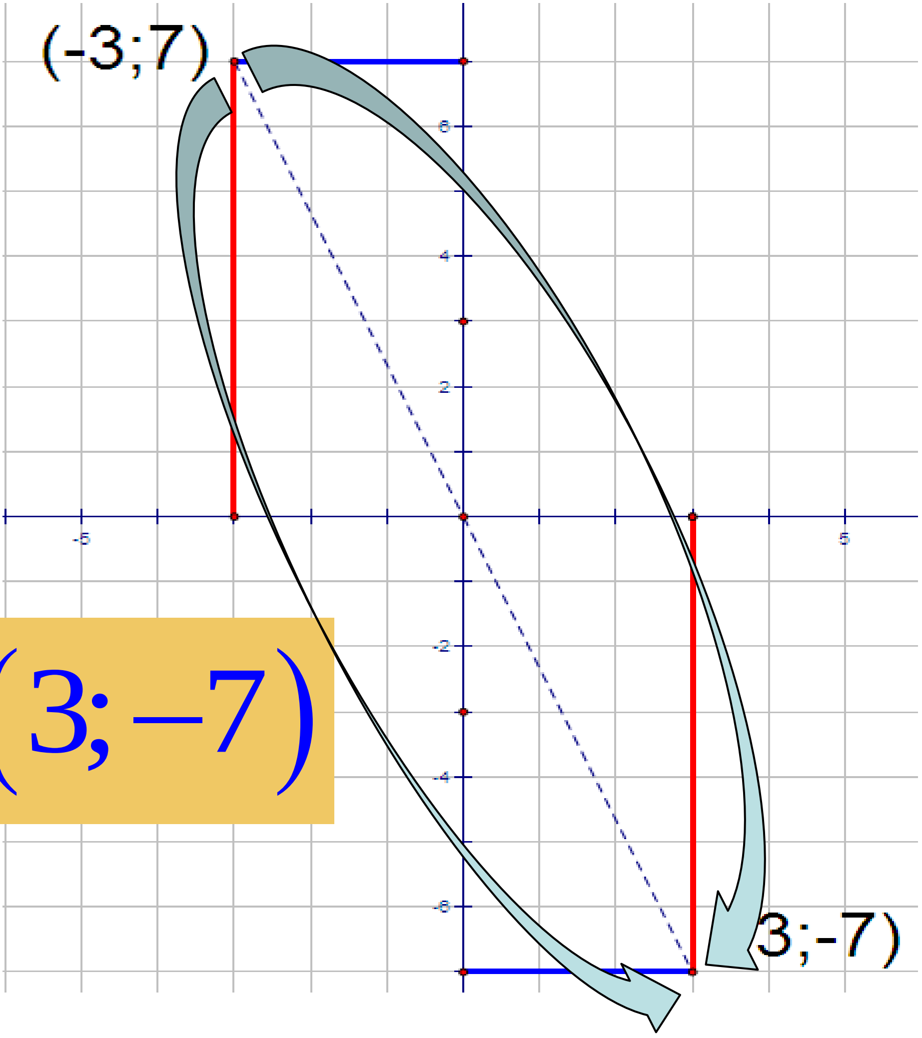
(b) $(3;-4)$ is rotated in a clockwise direction through $90°$.



$$(3;-4) \xrightarrow{-90°} (-4;-3)$$

Tutorial 2 Problem 1(c): Suggested Solution

(c) $(-3;7)$ is rotated through $\pm 180°$.



$$(-3;7) \xrightarrow{\pm 180°} (3;-7)$$

Tutorial 2: Part 2: Rotation about the origin

2. The vertices of quadrilateral $ABCD$ are $A(0;0)$; $B(1;3)$; $C(3;5)$ and $D(6;2)$. This quadrilateral is rotate... the origin anti-clockwise...

(a) Write down the c... of the transformed quadrilateral.

(b) Sketch the quadrilateral and the transformation thereof.

PAUSE DVD

- Do Tutorial 2 Part 2
- Then View Solutions

Tutorial 2 Problem 2(a): Suggested Solution

2. The vertices of quadrilateral $ABCD$ are $A(0;0)$; $B(1;3)$; $C(3;5)$ and $D(6;2)$. This quadrilateral is rotated about the origin through $90°$.

(a) Write down the co-ordinates of the transformed quadrilateral.

$$(0;0)\xrightarrow{90^\circ}(0;0)$$

$$(1;3)\xrightarrow{90^\circ}(-3;1)$$

$$(3;5)\xrightarrow{90^\circ}(-5;3)$$

$$(6;2)\xrightarrow{90^\circ}(-2;6)$$

Tutorial 2 Problem 2(b): Suggested Solution

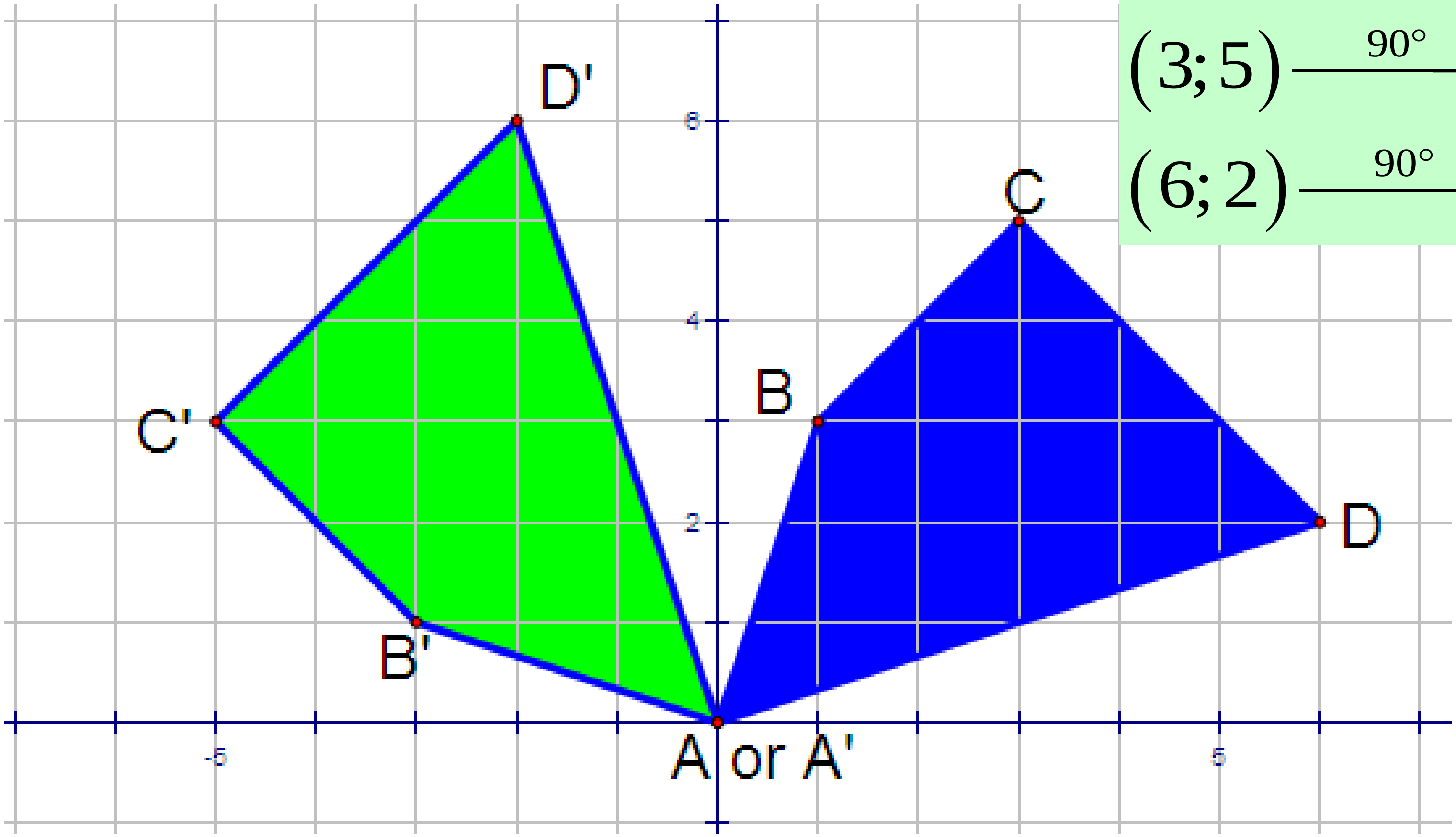
(b) Sketch the quadrilateral and the transformation thereof.

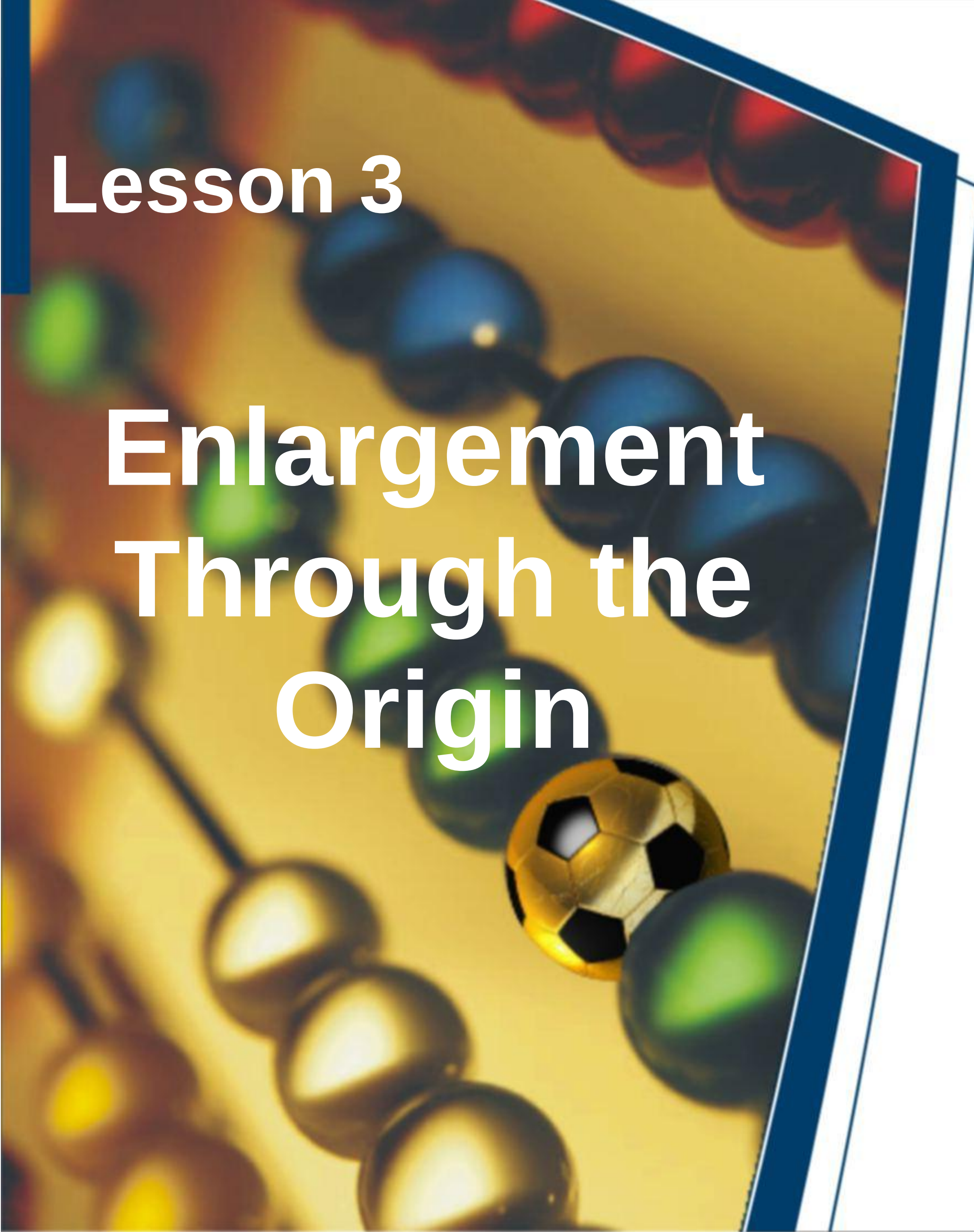
$$(0;0) \xrightarrow{90^\circ} (0;0)$$

$$(1;3) \xrightarrow{90^\circ} (-3;1)$$

$$(3;5) \xrightarrow{90^\circ} (-5;3)$$

$$(6;2) \xrightarrow{90^\circ} (-2;6)$$





Lesson 3

Enlargement Through the Origin

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Enlargement through the origin

$\Delta A'OB'$ is an enlargement of ΔAOB through the origin with a scale factor of 3.

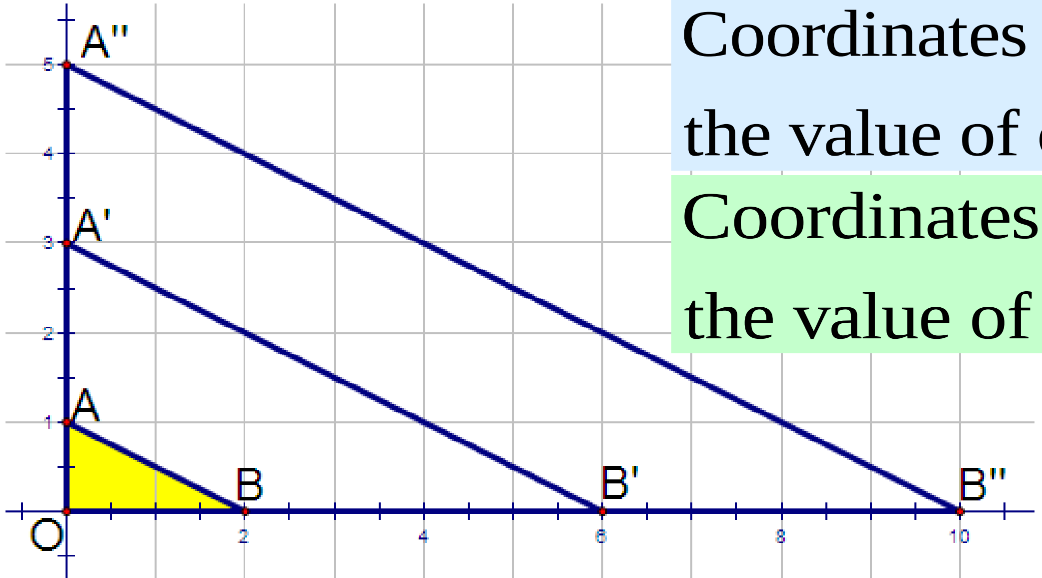
Area of $\Delta A'OB'$ $= 9\times$ Area of ΔAOB

$\Delta A''OB''$ is an enlargement of ΔAOB through the origin with a scale factor of 5.

Area of $\Delta A''OB''$ $= 25\times$ Area of ΔAOB

$$A'O : AO = B'O : BO = A'B' : AB = 3:1$$

$$A''O : AO = B''O : BO = A''B'' : AB = 5:1$$



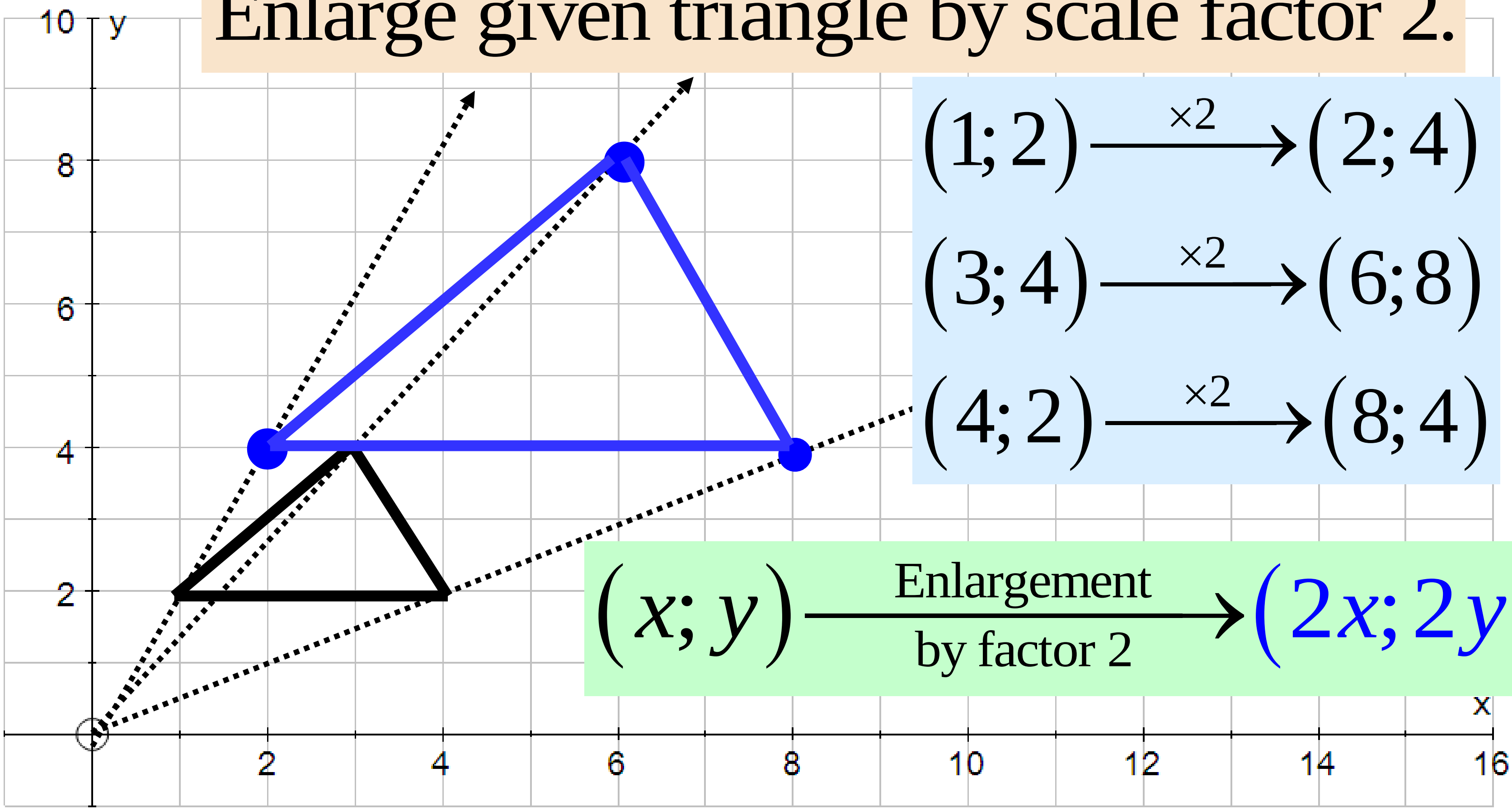
Coordinates of $\Delta A'OB'$ are three times the value of coordinates of ΔAOB.

Coordinates of $\Delta A''OB''$ are five times the value of coordinates of ΔAOB.

Enlargement when one vertex is not on the origin

Strategy: Draw a line through the origin and each vertex in order to find the enlargements.

Enlarge given triangle by scale factor 2.



$$(1;2)\xrightarrow{\times 2}(2;4)$$

$$(3;4)\xrightarrow{\times 2}(6;8)$$

$$(4;2)\xrightarrow{\times 2}(8;4)$$

$$(x;y)\xrightarrow[\text{by factor 2}]{\text{Enlargement}}(2x;2y)$$

Tutorial 3: Enlargement through the origin

Given a square $ABCD$ with vertices $A(1;2)$; $B(1;1)$; $C(2;1)$ and $D(2;2)$.

$ABCD$ is enlarged by the factors 3 and 5 to obtain the transformed squares $A'B'C'D'$ and $A''B''C''D''$ respectively.

1. Write down the coordinates of the two enlarged squares.
2. Sketch the original and the two enlarged squares on the same system of axes.
3. Complete the following:
   (a) $AB : A'B' = \cdots$
   (b) $BC : B''C'' = \cdots$
   (c) $A'D' : A''D'' = \cdots$
4. Compare the areas of the three squares.

PAUSE DVD

- Do Tutorial 3
- Then View Solutions

Tutorial 3 Problem 1: Suggested Solution

Given a square $ABCD$ with vertices $A(1;2)$; $B(1;1)$; $C(2;1)$ and $D(2;2)$.

$$ABCD \xrightarrow{\text{Enlarged by factor 3}} A'B'C'D'$$

$$ABCD \xrightarrow{\text{Enlarged by factor 5}} A''B''C''D''$$

1. Write down the coordinates of the two enlarged squares.

$A'(3;6)$

$B'(3;3)$

$C'(6;3)$

$D'(6;6)$

$A''(5;10)$

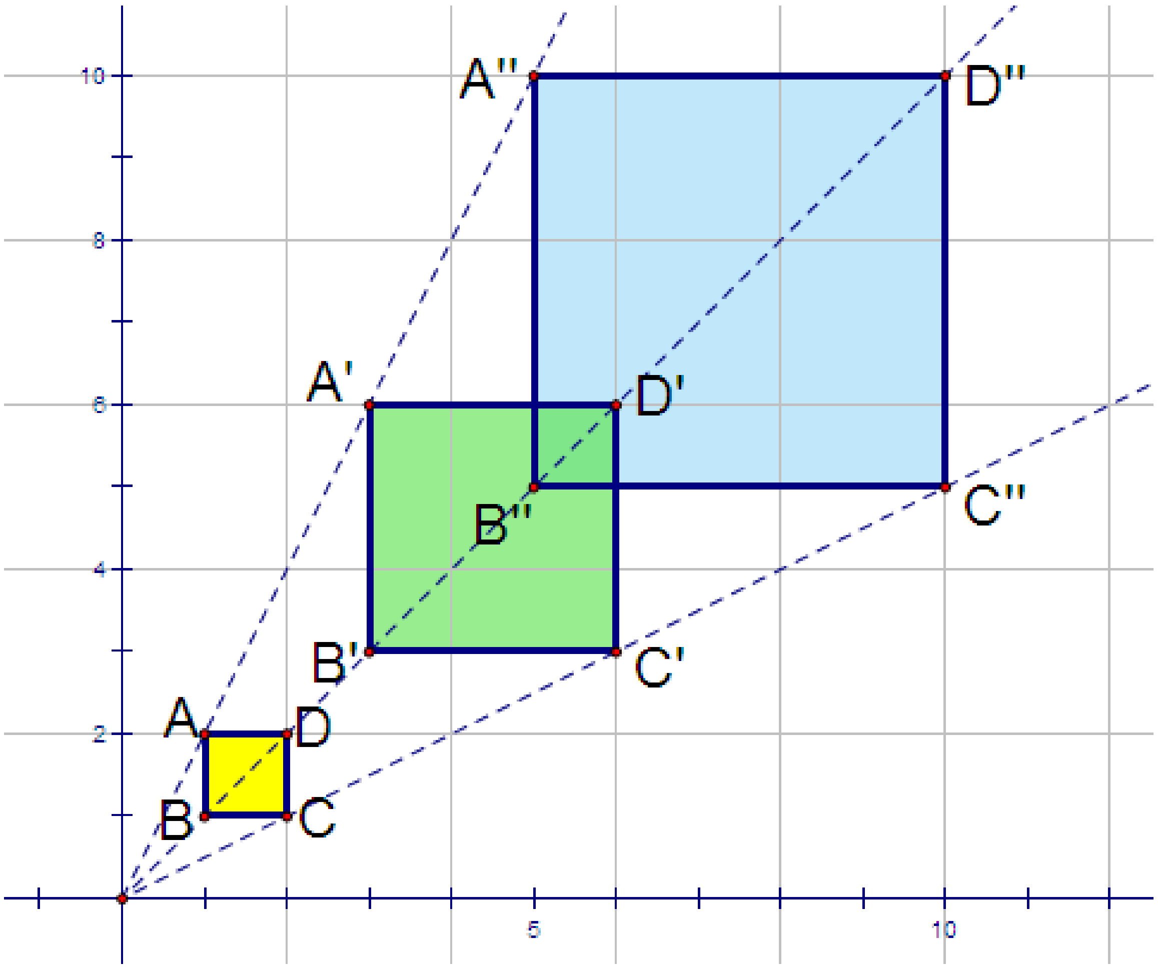
$B''(5:5)$

$C''(10;5)$

$D''(10;10)$

Tutorial 3 Problem 2: Suggested Solutions

2. Sketch the three squares.



$A'(3;6)$

$B'(3;3)$

$C'(6;3)$

$D'(6;6)$

$A''(5;10)$

$B''(5:5)$

$C''(10;5)$

$D''(10;10)$

Tutorial 3 Problem 3: Suggested Solution

3. Complete the following:

(a) $AB : A'B' = \cdots$

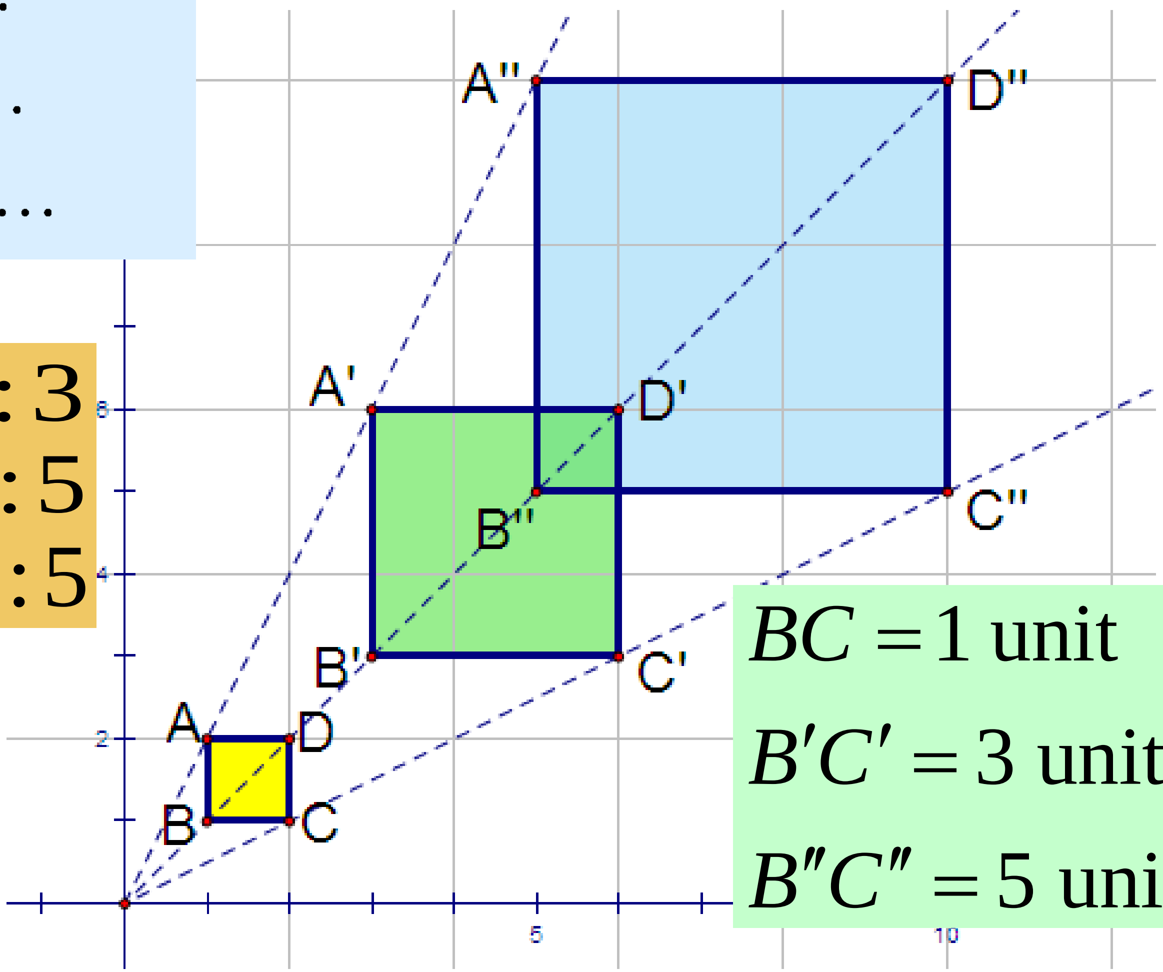
(b) $BC : B''C'' = \cdots$

(c) $A'D' : A''D'' = \cdots$

$AB : A'B' = 1:3$

$BC : B''C'' = 1:5$

$A'D' : A''D'' = 3:5$



$BC = 1$ unit

$B'C' = 3$ units

$B''C'' = 5$ units

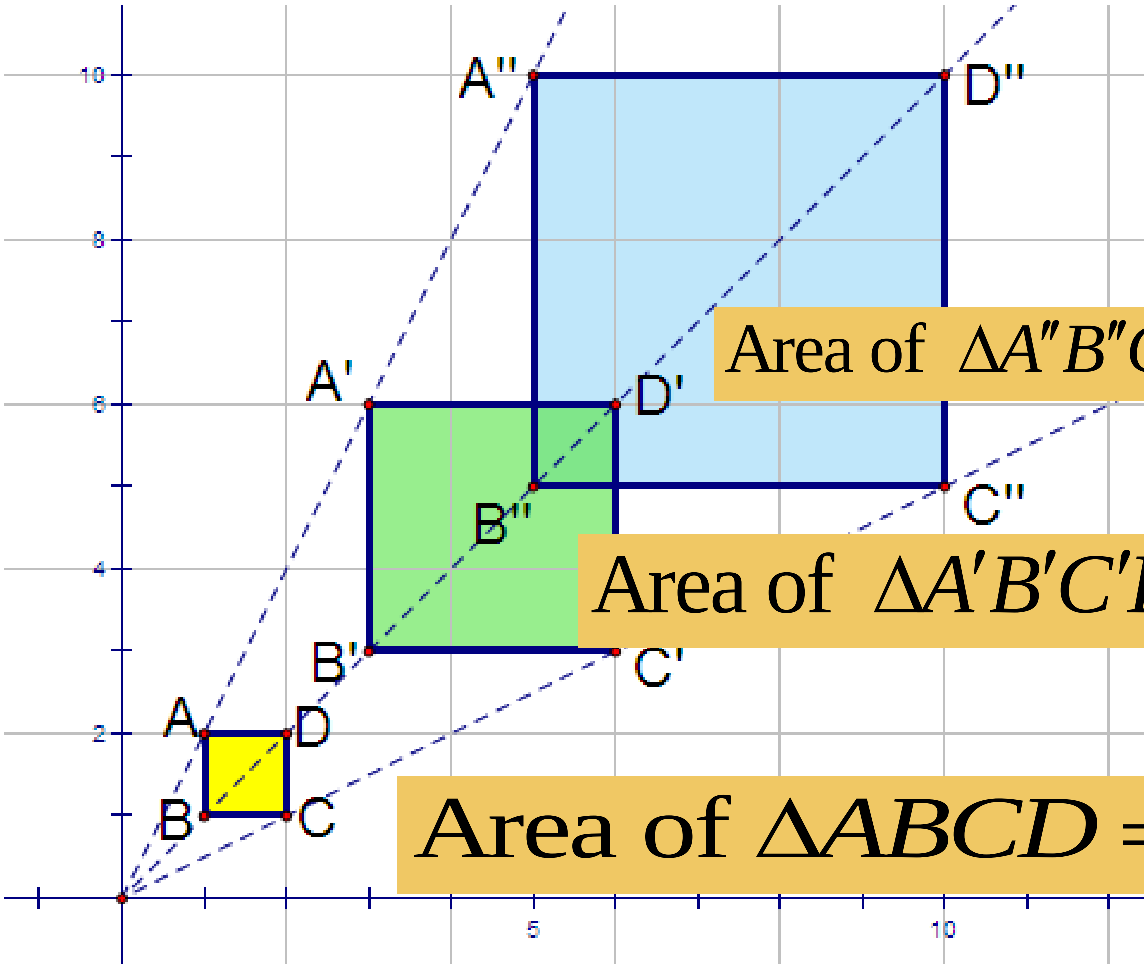
Tutorial 3 Problem 4: Suggested Solution

4. Compare the areas of the three squares.

$BC = 1$ unit

$B'C' = 3$ units

$B''C'' = 5$ units



Area of $\Delta A''B''C''D'' = 25$ square units

Area of $\Delta A'B'C'D' = 9$ square units

Area of $\Delta ABCD = 1$ square unit

End of the DVD on Transformation Geometry I

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in the DVD.
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!