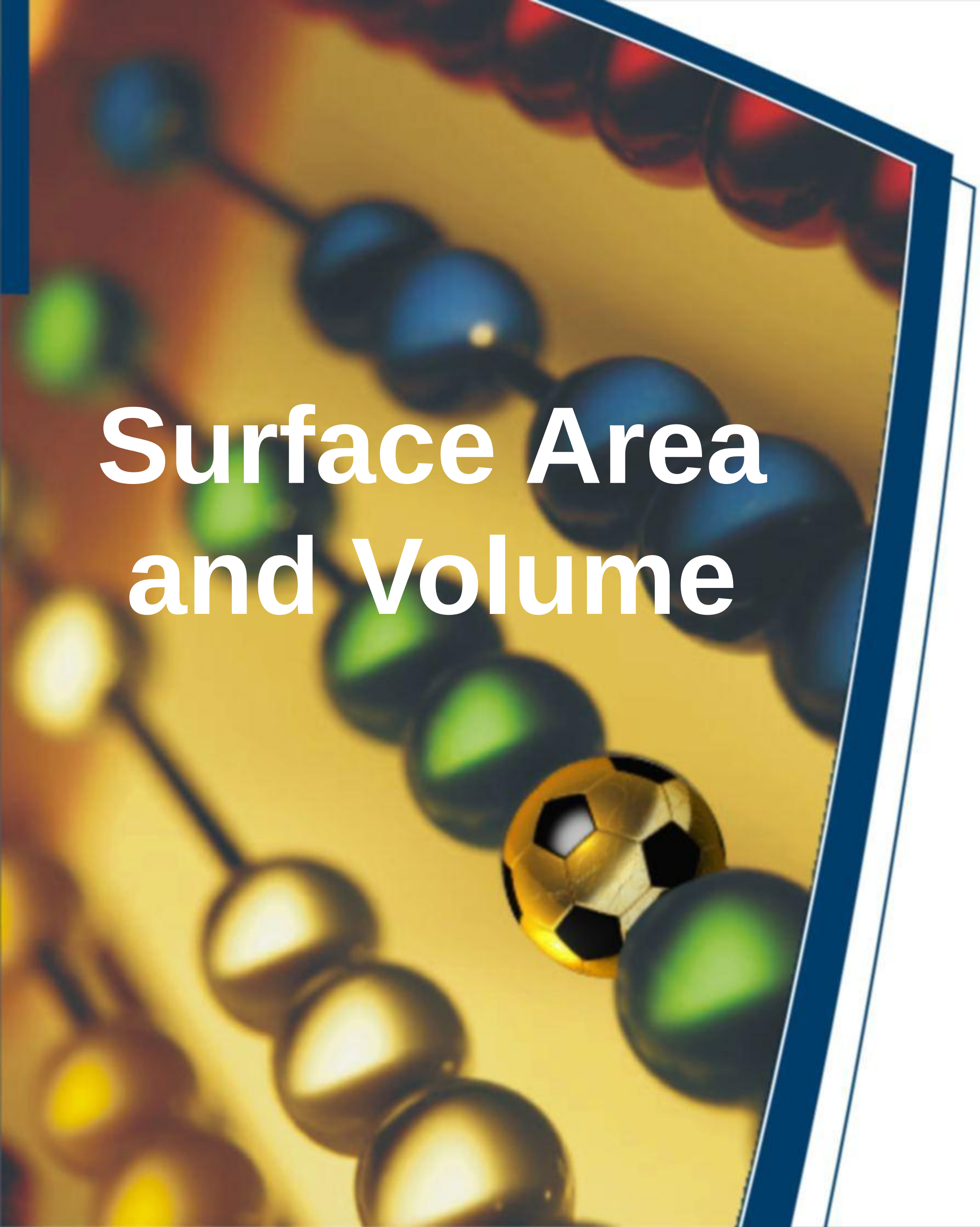

Surface Area and Volume

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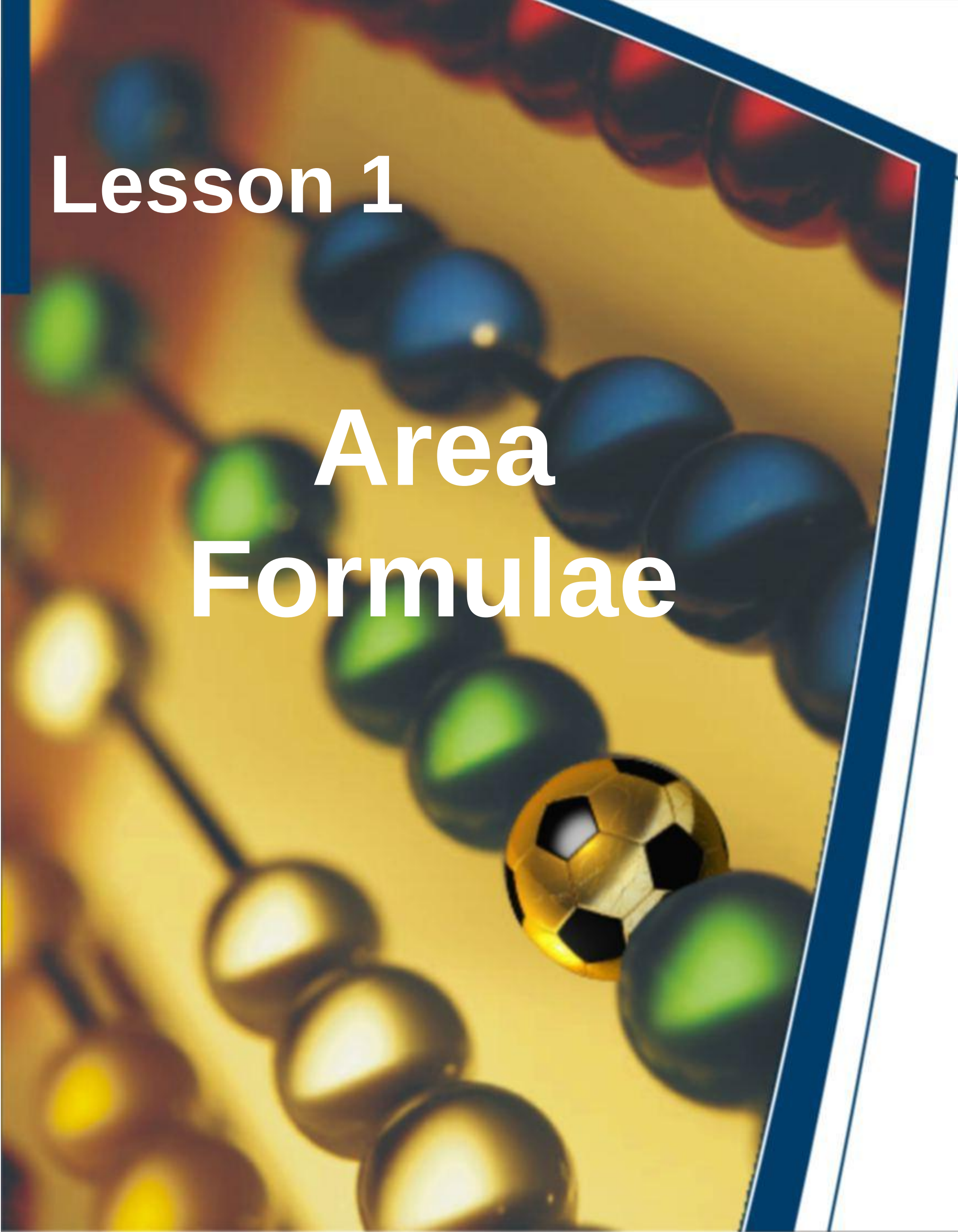
Outcomes for this DVD

In this DVD we will:

- Recall and apply **area formulae** LESSON 1.
- Recall and apply **volume formulae** LESSON 2.
- Apply area and volume formulae to **composite objects** LESSON 3.

Lesson 1

Area Formulae



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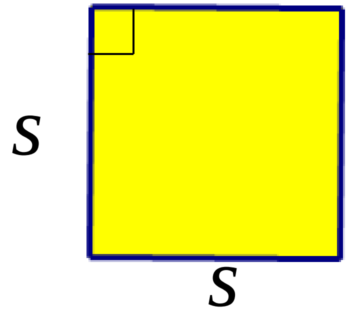
Area formulae: Square, Rectangle and Triangle

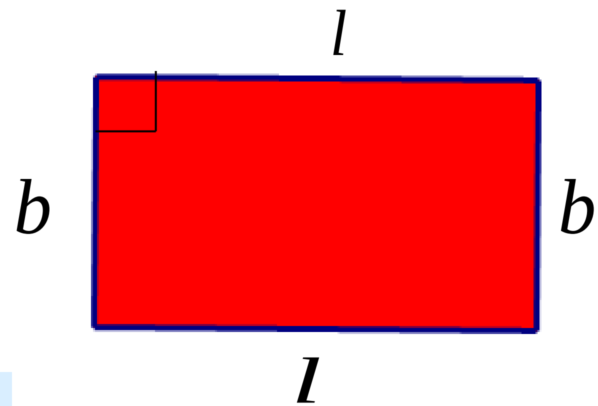
Square of side s

Area of a square $= s^2$



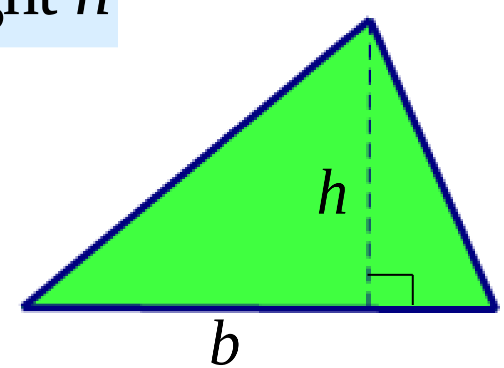
Rectangle with length l and breadth b

Area of Rectangle $= lb$



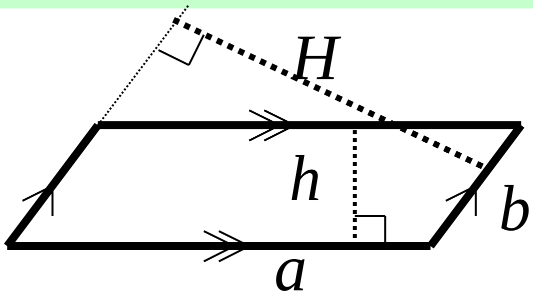
Area of triangle with base b and height h

Area of Triangle $= \dfrac{1}{2}bh$



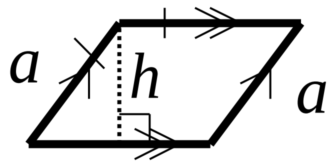
Area formulae: Parallelogram, Rhombus and Kite

You should know when a Quadrilateral is a Parallelogram.



$$\text{Area of a } ||^{\text{m}} = ah = bH$$

Rhombus is a $||^{\text{m}}$ with a adjacent sides equal.

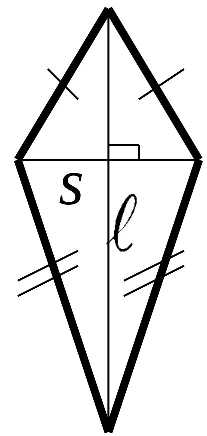


$$\text{Area of Rhombus} = ah$$

Properties of a Kite:

$$\text{Area of Kite} = 2 \times \left(\frac{1}{2} \ell \times \frac{s}{2} \right) = \frac{1}{2} \ell s$$

$$= \frac{1}{2} \times \text{ Product of Diagonals}$$



$s \perp \ell$

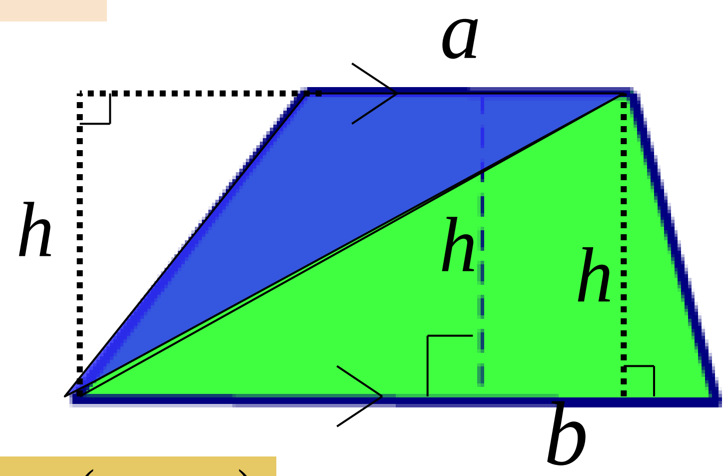
ℓ bisects s

Area formulae: Trapezium and Circle

Area of trapezium with two parallel sides *a* and *b* and height *h*

$$\text{Area of Blue } \Delta = \frac{1}{2} \times a \times h = \frac{ah}{2}$$

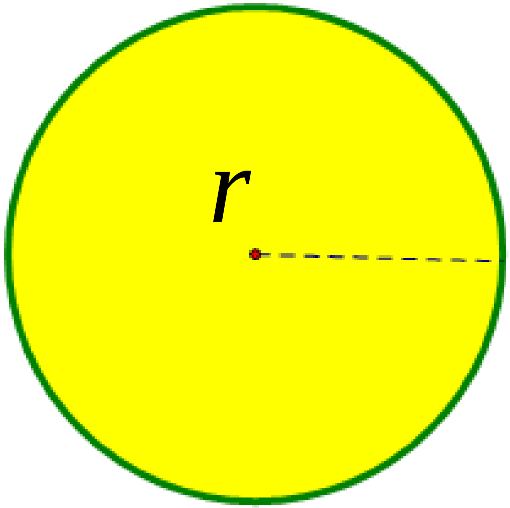
$$\text{Area of Green } \Delta = \frac{1}{2} \times b \times h = \frac{bh}{2}$$



$$\text{Area of Trapezium} = \frac{ah}{2} + \frac{bh}{2} = \frac{h(a+b)}{2}$$

Area of circle with radius *r*

$$\text{Area of } \odot = \pi r^2$$



Example: Surface Area of Right Pyramid

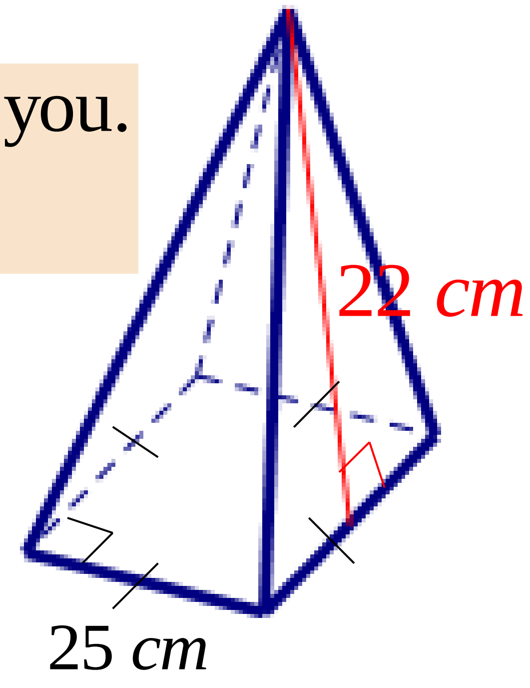
The total surface area (TSA) of a 3-dimensional object can be found by **totalling the areas** of each of the **shapes that make up the outside** of the object.

Find the total surface area of the right pyramid.

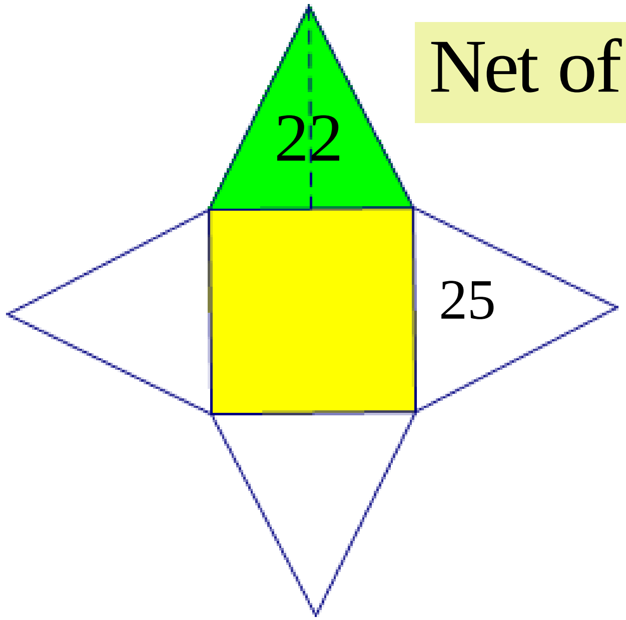
Draw a 2 – D net of the pyramid to help you.

Round your answer to 2 decimal places.

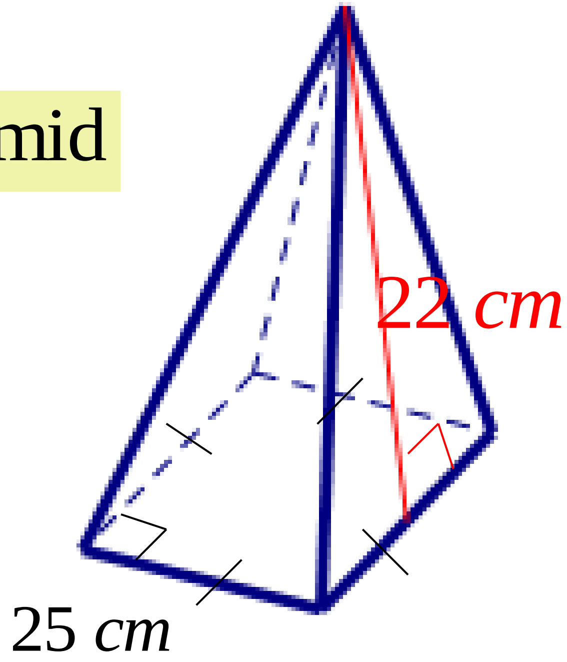
<u>Note</u>: A net is a 2 – D representation of the faces (surface areas) of the 3 – D object.



Solution: Surface area of Right Pyramid



Net of our right pyramid



Surface area of right pyramid

$$= \text{Area of base (square)} + 4 \times \text{area of triangle} = l^2 + 4 \times \left(\frac{1}{2} bh\right)$$

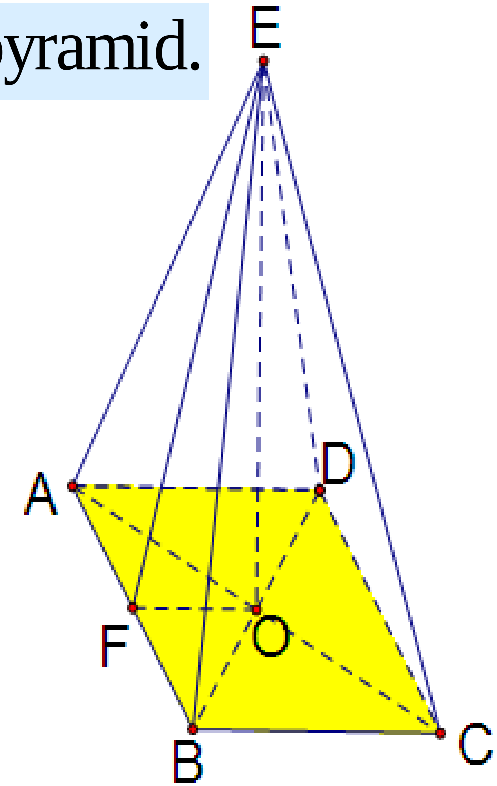
$$= \left[25 \times 25 + 4\left(\frac{1}{2} \times 25 \times 22\right)\right] cm^2 = (625 + 1100)\ cm^2 = 1725\ cm^2$$

Tutorial 1: Surface Area of a Pyramid

$ABCDE$ is a regular pyramid of which the base $ABCD$ is a square.

$FE \perp AB$ is the lateral (or slant) height of the pyramid.

$EO \perp$ base is the height of the pyramid.



1) If $AB = 8\ cm$ and $FE = 12\ cm$.
   (a) Draw a net of the pyramid.
   (b) Calculate the total surface area of this pyramid.
2) Calculate the total surface area of such a pyramid if $BC = 12\ cm$ and $EO = 8\ cm$.
3) If $FE = l$ and $AB = b$, show that the total surface area for any such pyramid is given by $b(2l + b)$.

PAUSE DVD

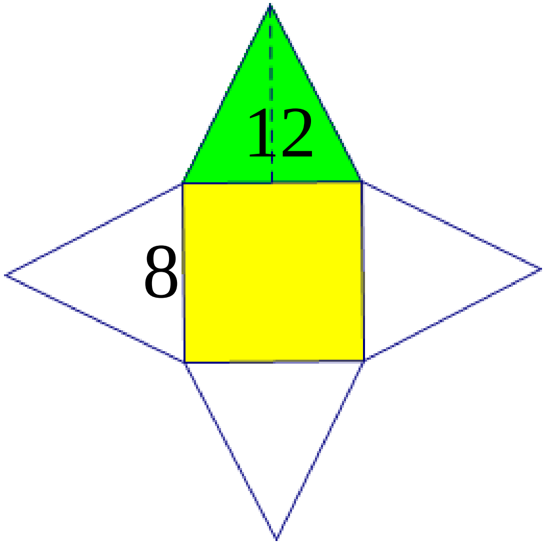
- Do Tutorial 1
- Then View Solutions

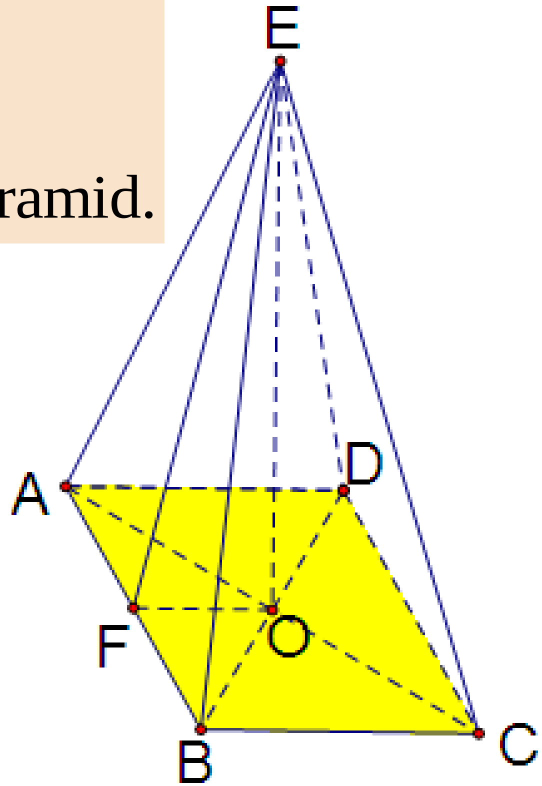
Tutorial 1 Problem 1: Suggested Solution

1) If $AB = 8\ cm$ and $FE = 12\ cm$.

(a) Draw a net of the pyramid.

(b) Calculate the total surface area of this pyramid.





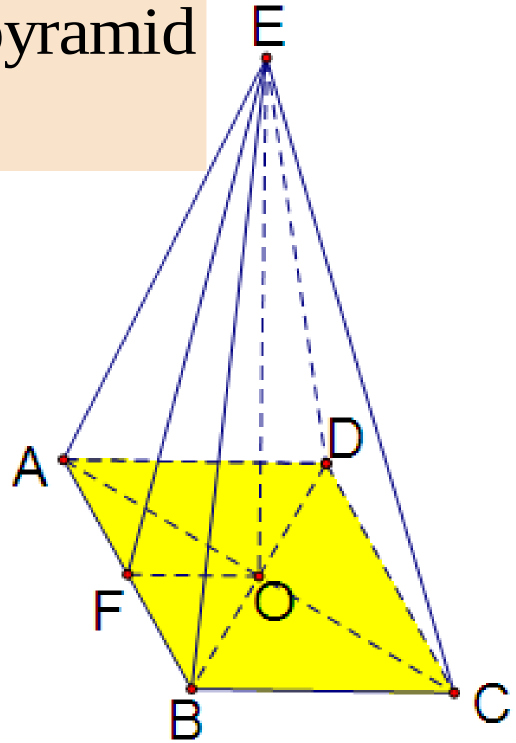
Total surface area

$= \text{ area of base } + \ 4 \times \text{area of triangle}$

$$= \left[8\times 8 + 4\times \frac{1}{2}\times 8\times 12 \right] = (64+192) = 256\ cm^2$$

Tutorial 1 Problem 2: Suggested Solution

2) Calculate the total surface area of such a pyramid if $BC = 12\ cm$ and $EO = 8\ cm$.



$$\text{Slanted height } = FE = \sqrt{FO^2 + EO^2}$$

$$\therefore FE = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10\ cm$$

$\therefore$ Total Surface Area

$$= 12 \times 12 + 4 \times \frac{1}{2} \times 12 \times 10 = 144 + 240 = 384\ cm^2$$

Tutorial 1 Problem 3: Suggested Solution

3) If $FE = l$ and $AB = b$, show that the total surface area for any such pyramid is given by $b(2l + b)$.

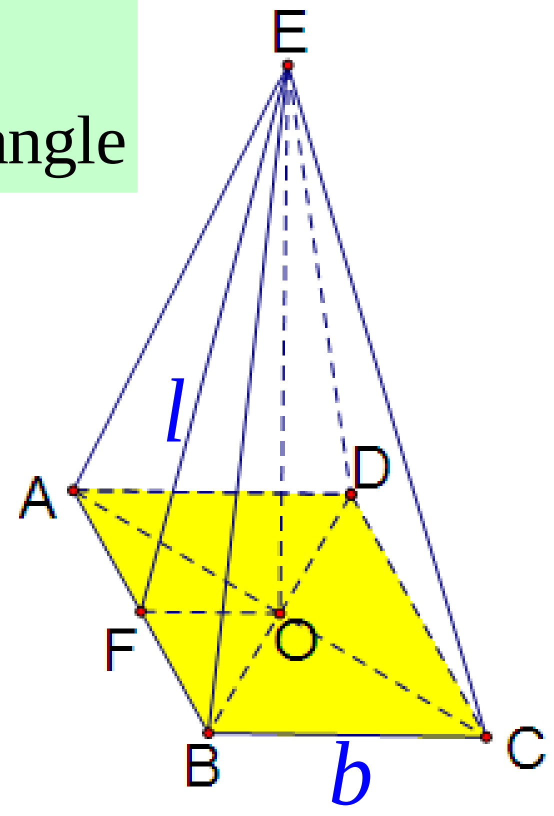
Total Surface Area

$=$ Area of Base $+$ 4 Area of Slanted Triangle

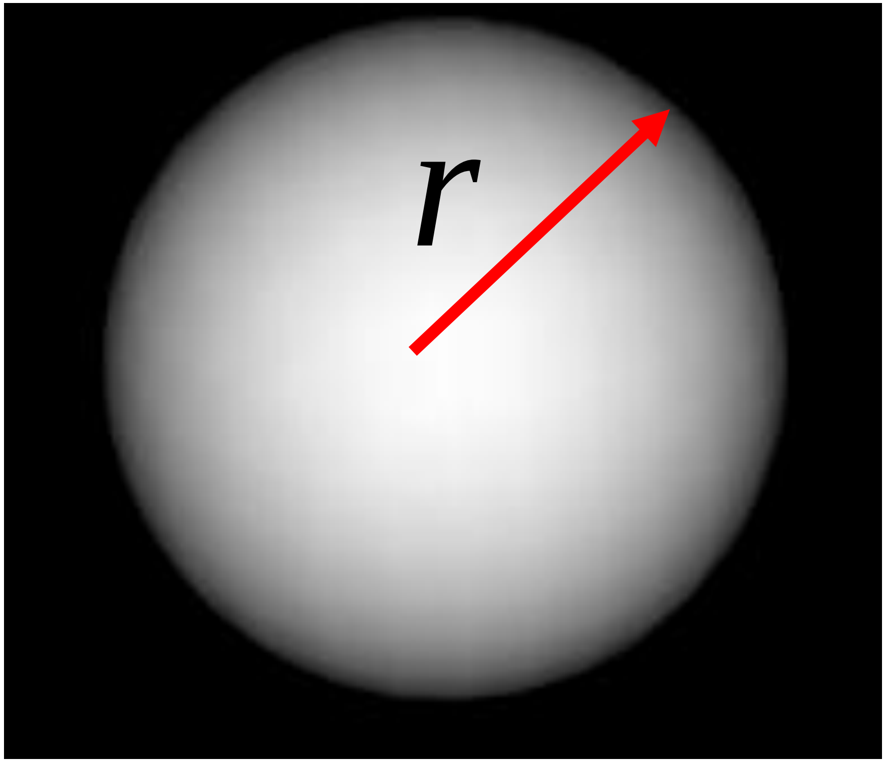
$$= b^2 + 4\left(\frac{1}{2}bl\right)$$

$$= b^2 + 2bl$$

$$= b(b + 2l)$$



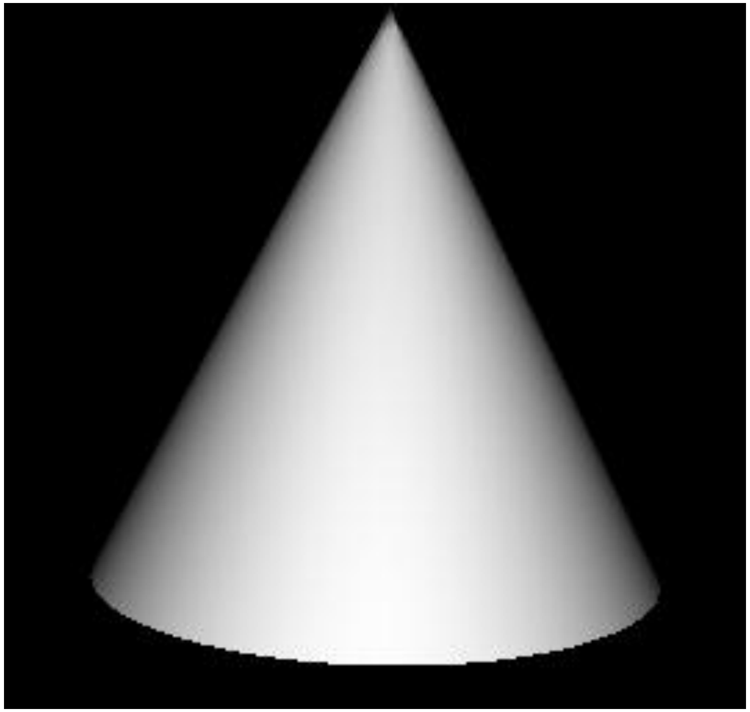
Surface Area of a Sphere

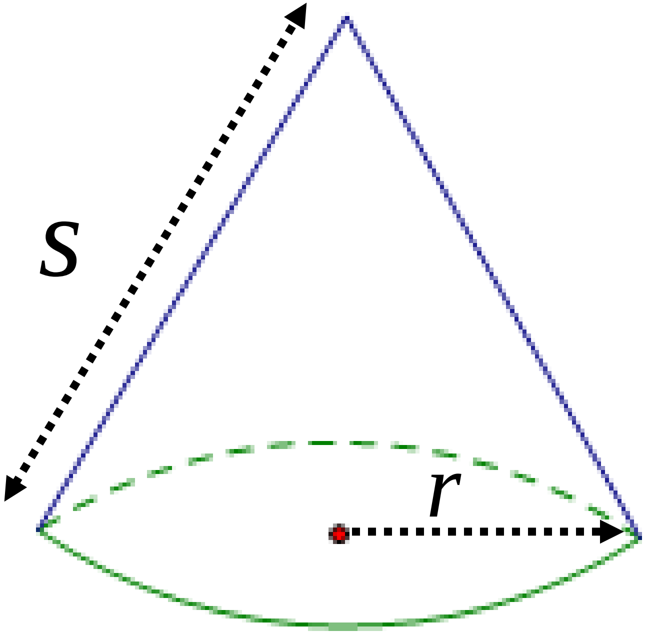


Surface area of a sphere of radius *r*

$$A = 4\pi r^2$$

Surface Area of Cones



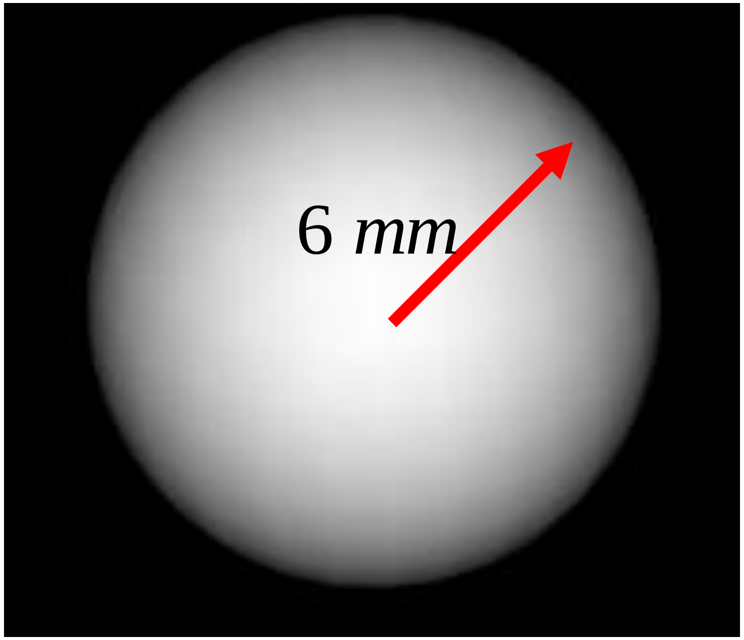


Surface area of a cone with radius r and slant height s:

$$A = \pi r^2 + \pi r s$$
$$= \pi r (r + s)$$

Example: Surface Area of a Sphere

Find the surface area of a sphere with radius 6 mm.



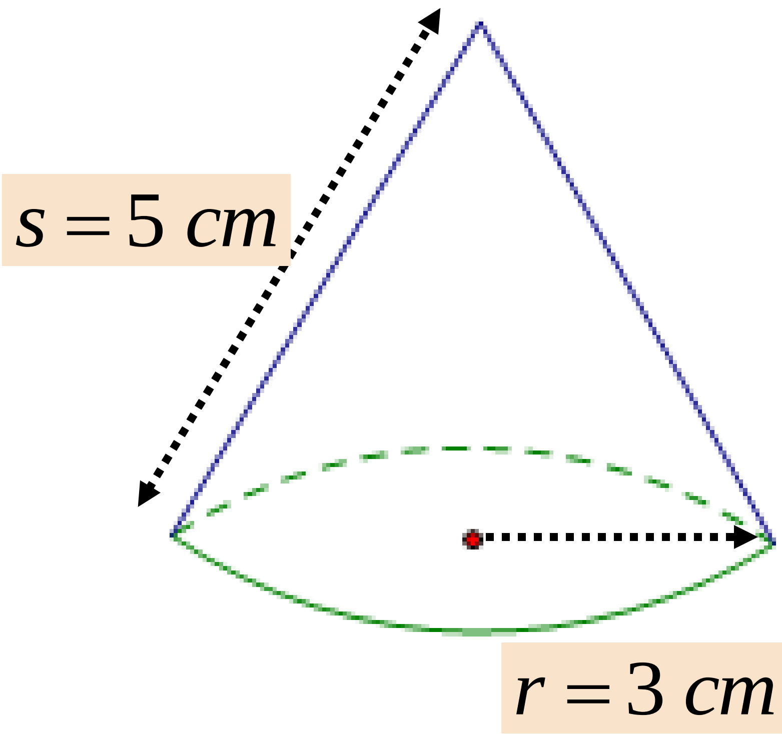
$$\text{Surface area } = 4\pi r^2$$

$$= 4 \times \pi \times (6\ mm)^2$$

$$= 452,39\ mm^2$$

Example: Surface Area of Cone

Find the surface area of a cone with radius 3 *cm* slant height 5 *cm*.



$$A = \pi r(r + s)$$

$$= \pi \times 3(3+5)\ cm^2$$

$$= 75{,}398\ cm^2$$

Tutorial 2: Surface areas of Cones and Spheres

1. Calculate the surface areas of:

   (a) a sphere with radius 8,5 cm

   (b) a sphere with diameter $\frac{\pi}{2}m$

2. Find the total surface areas of the cones below:

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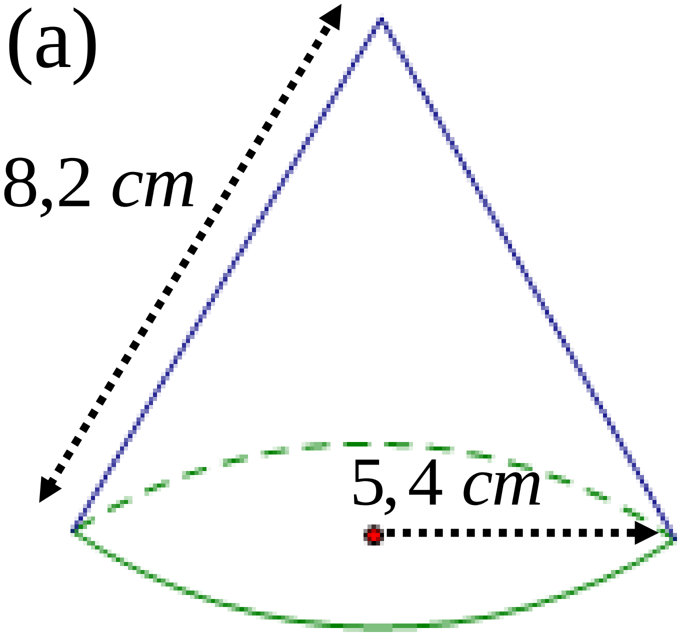
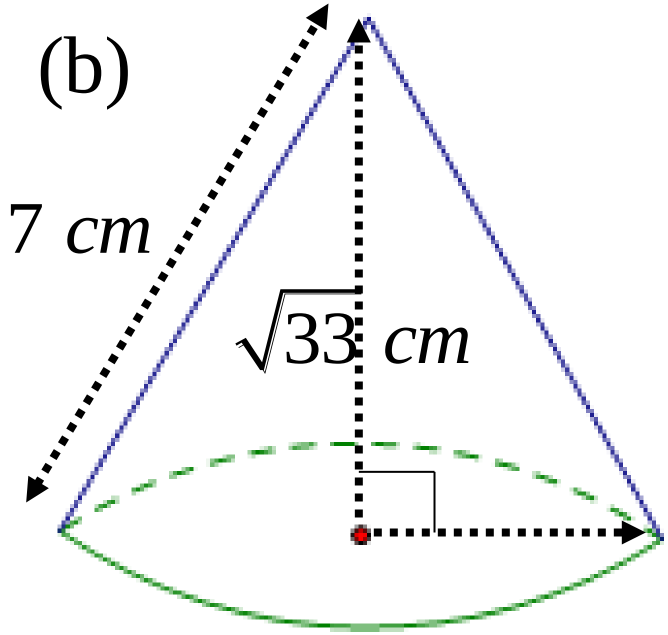
- Do Tutorial 2
- Then View Solutions

(a)



(b)



Tutorial 2 Problem 1: Suggested Solution

1. Calculate the surface areas of:

   (a) A sphere with radius 8,5 cm.

   (b) A sphere with diameter $\frac{\pi}{2}\ m$.

1. (a) Surface area of sphere

$$= 4\pi r^2$$

$$= 4 \times \pi \times (8,5\ cm)^2 = 907,92\ cm^2$$

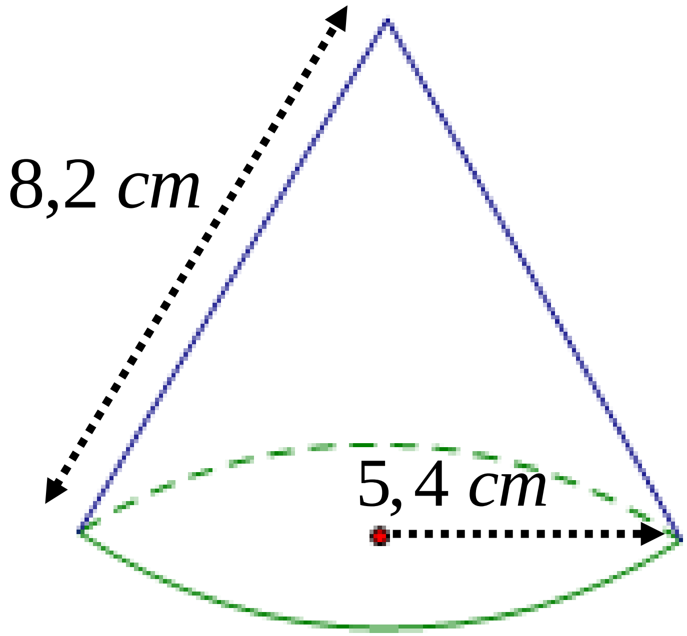
1. (b) Surface area of sphere

$$= 4\pi r^2$$

$$= 4 \times \pi \times \left(\frac{\pi}{4} m \right)^2 = 7,75\ m^2$$

Tutorial 2 Problem 2(a): Suggested Solution

2. (a) Find the total surface area of the cone below:



<u>Given</u>: $r = 5,4\ cm$

$s = 8,2\ cm$

Surface area of cone

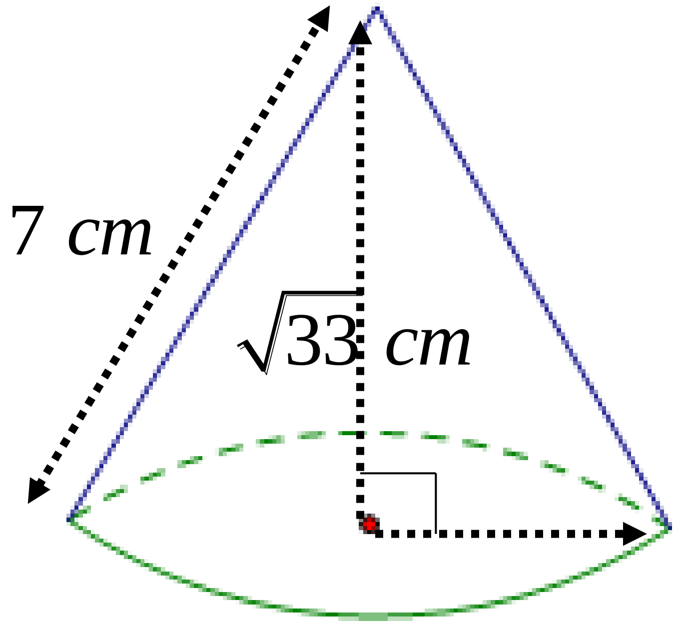
$= \pi r \left(r + s \right)$

$= \pi \times 5,4 \times \left(5,4 + 8,2\right)\ cm^2$

$= 230,72\ cm^2$

Tutorial 2 Problem 2(b): Suggested Solution

2. (b) Find the total surface area of the cone below:



<u>Given:</u>

$h = \sqrt{33}\ cm$

$s = 7\ cm$

$$r = \sqrt{s^2 - h^2}$$

$$= \sqrt{49 - 33}\ cm$$

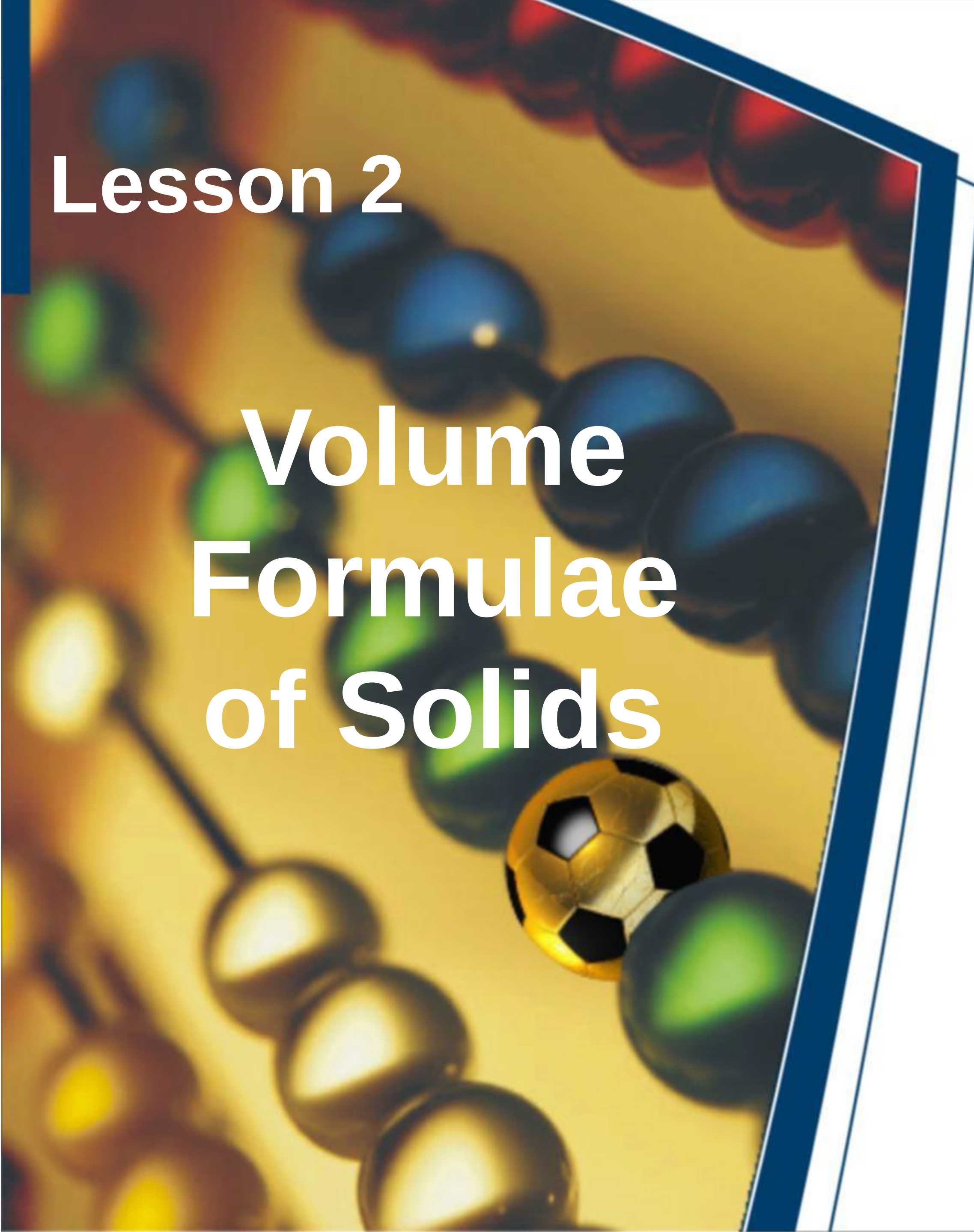
$$= 4\ cm$$

Surface area of cone

$$= \pi r(r + s)$$

$$= \pi \times 4 \times (4 + 7)\ cm^2$$

$$= 138,23\ cm^2$$



Lesson 2

Volume Formulae of Solids

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Volume of any Prism

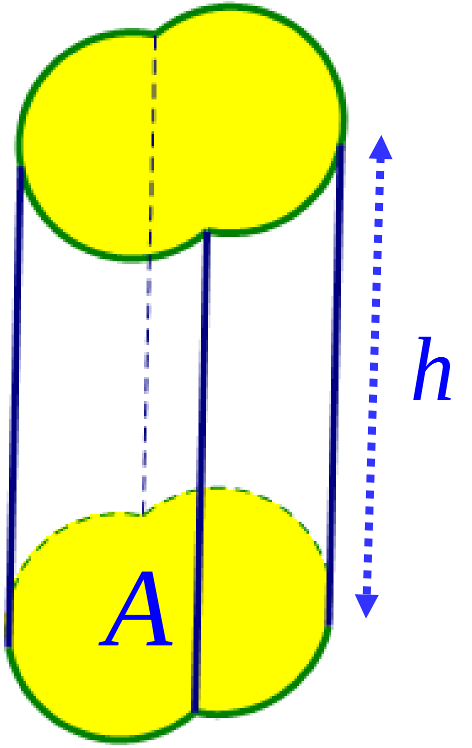
The volume of any prism can be found by using its cross-sectional area:

Volume of any prism

= area of cross sectional area × height (or length)

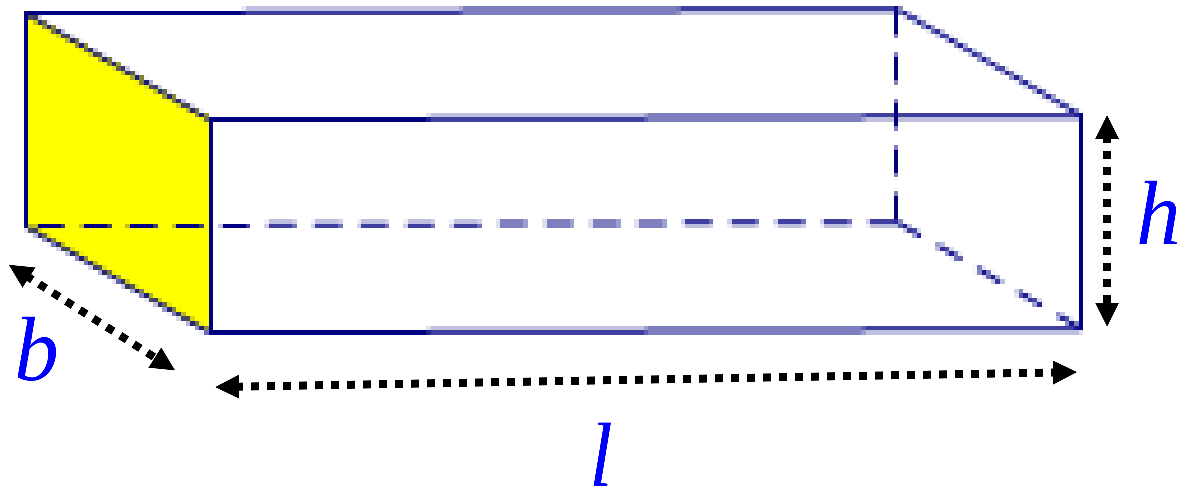
= area of base × height

$$V = Ah$$



Volume of Rectangular Prism

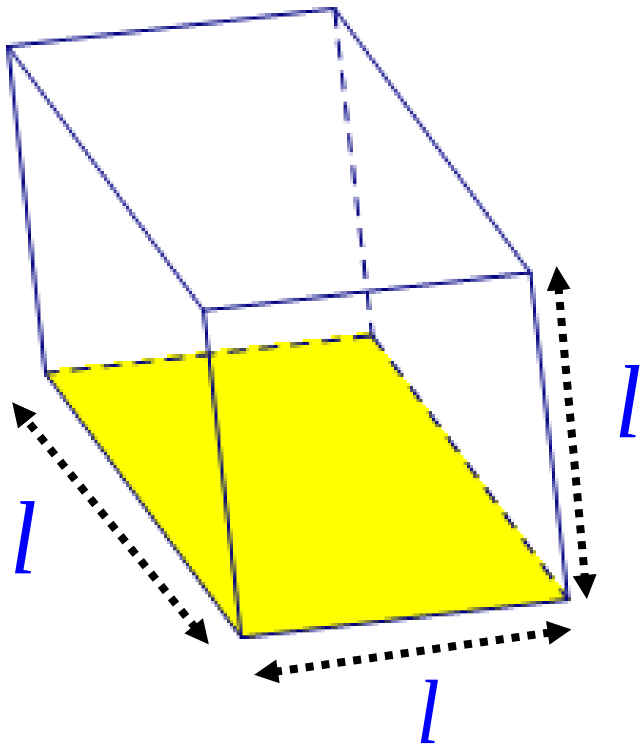
Rectangular prism (cuboid)



$$V = lbh$$

Volume of Square Prism

Square prism (cube)

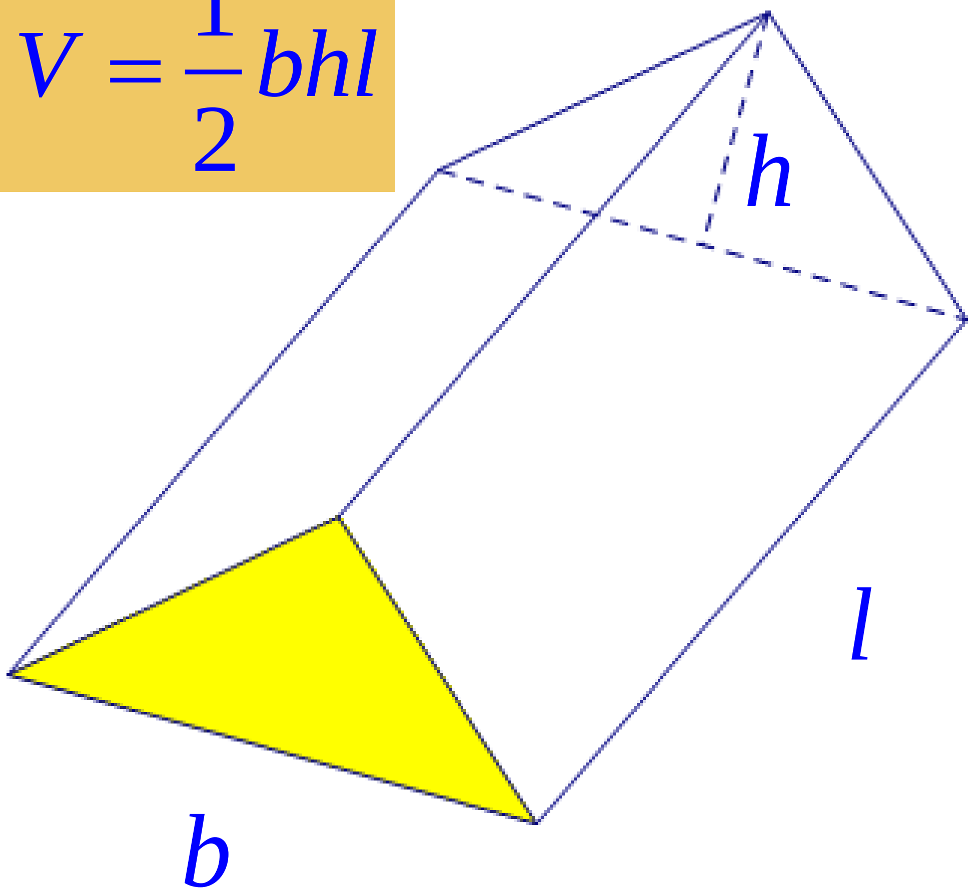


$$V = l^3$$

Volume of Triangular Prism

$$V = \text{Base Area} \times \text{Height} = \left(\frac{1}{2}bh\right) \times l$$

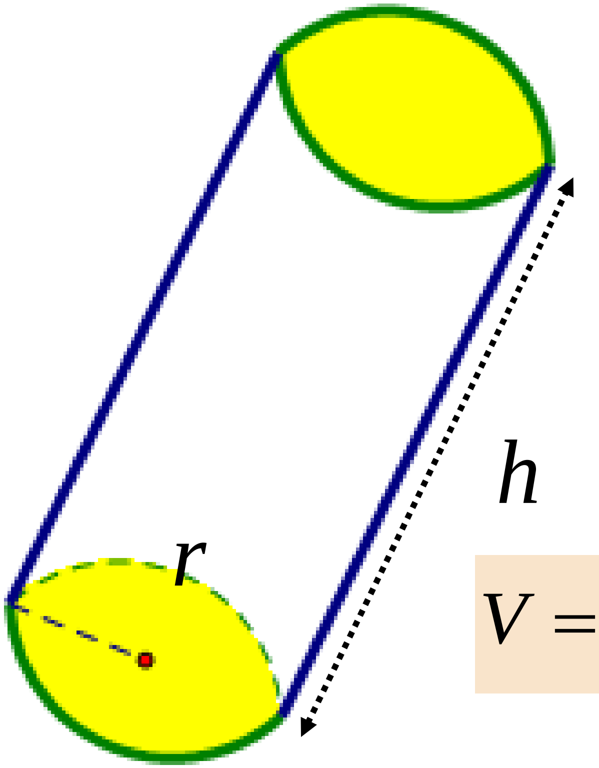
$$\text{Triangular Prism: } V = \frac{1}{2}bhl$$



Volume of Circular Prism

Circular Prism (Cylinder)

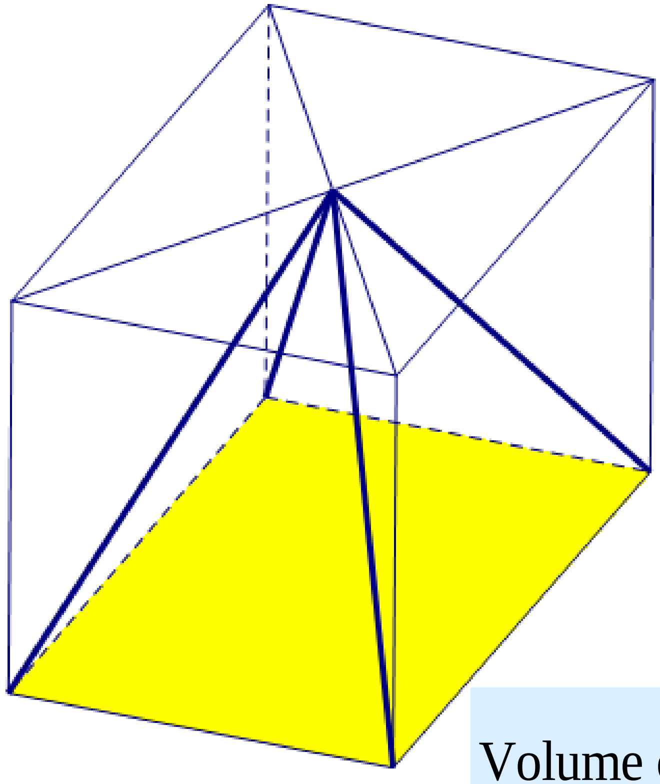
$$V = \pi r^2 h$$



$$V = \text{Base Area} \times \text{Height} = \left(\pi r^2\right) \times h$$

Volumes of Right Pyramids

A right pyramid can fit inside a prism as shown in the diagram:



It can be proved that the pyramid occupies one-third the volume of the prism containing it.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{the volume of prism}$$

$$= \frac{1}{3} \times \text{base area} \times \text{ perpendicular height}$$

Tutorial 3: Volume of Pyramids

1. Find the volumes of the following square pyramids with base sides and heights of:
   (a) 4 cm (b) 9,5 cm

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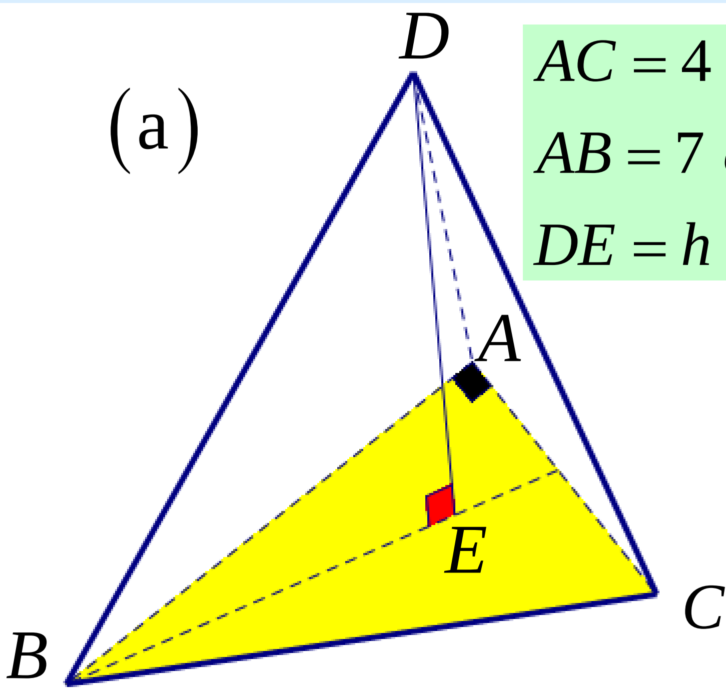
- Do Tutorial 3
- Then View Solutions

2. Calculate the volume of the following pyramids, correct to 2 decimal places.

(a)

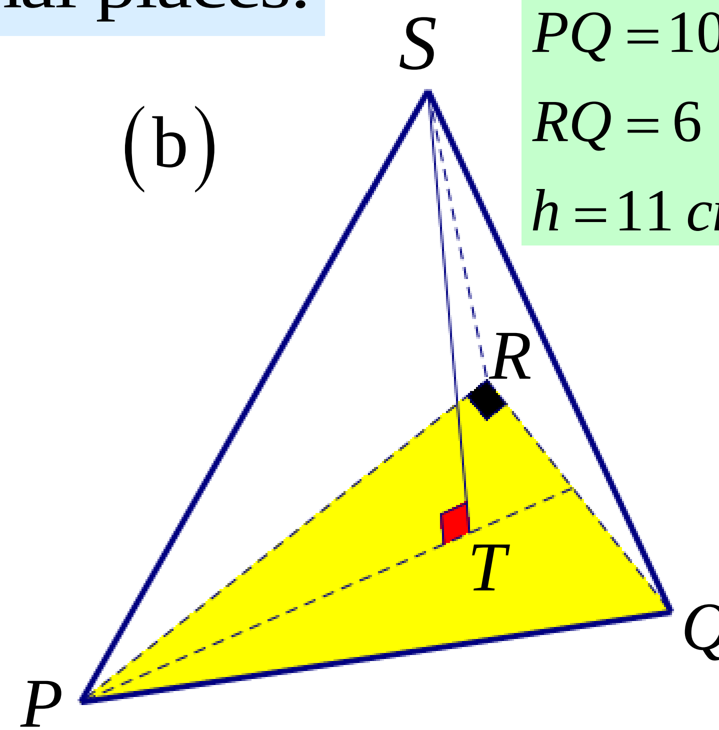


$AC = 4\ cm$

$AB = 7\ cm$

$DE = h = 6\ cm$

(b)



$PQ = 10\ cm$

$RQ = 6\ cm$

$h = 11\ cm$

Tutorial 3 Problem 1: Suggested Solutions

1. Find the volumes of the following square pyramids with base sides and heights of:

   (a) 4 cm (b) 9,5 cm

$$1.(a)\quad \text{Volume} = \frac{1}{3}l^3$$

$$= \frac{1}{3}(4\ cm)^3$$

$$= 21\tfrac{1}{3}\ cm^3$$

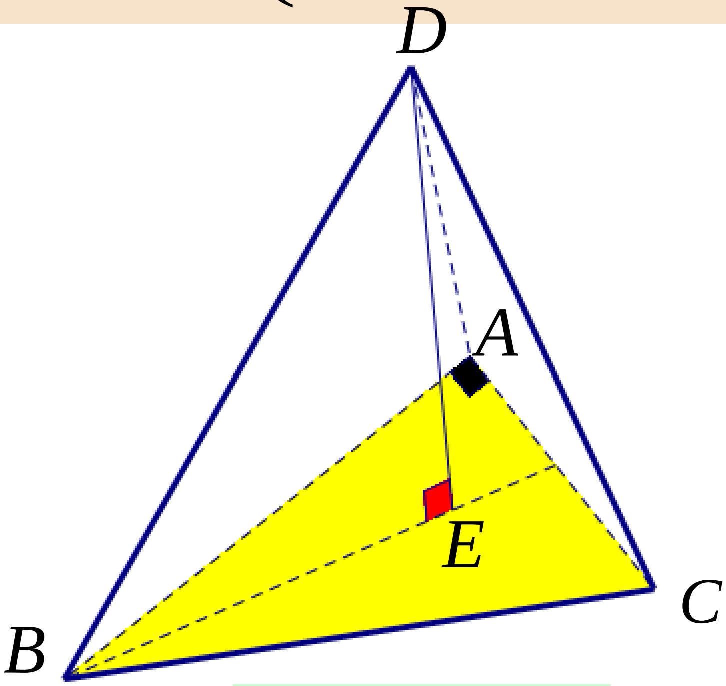
$$1.(b)\quad \text{Volume} = \frac{1}{3}l^3$$

$$= \frac{1}{3}(9{,}5\ cm)^3$$

$$= 285{,}79\ cm^3$$

Tutorial 3 Problem 2(a): Suggested Solution

2.(a) Calculate the volume of the pyramid. (Correct to 2 decimal places)



$AC = 4\ cm$

$AB = 7\ cm$

$DE = h = 6\ cm$

Volume of pyramid

$$= \frac{1}{3} \times \text{ base area } \times \text{ height}$$

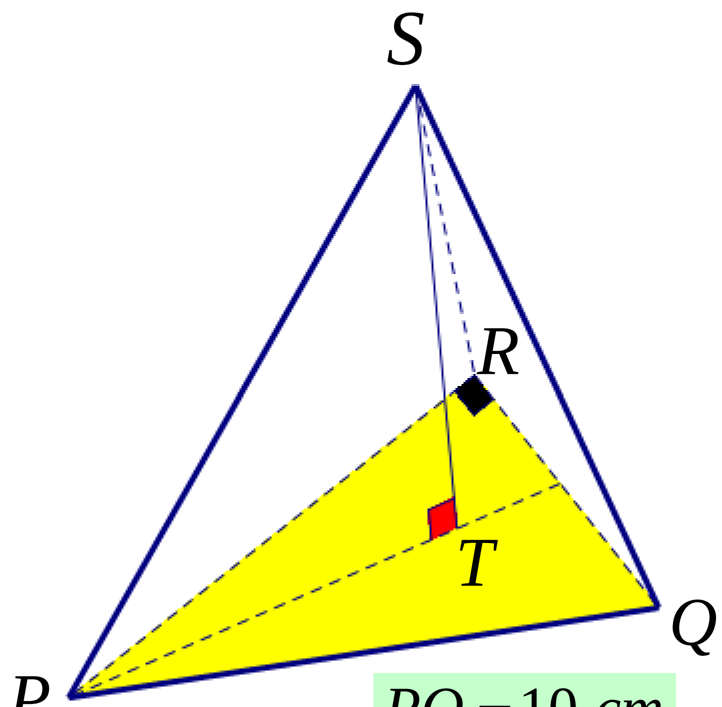
$$= \frac{1}{3} \times \left(\frac{1}{2} \times AB \times AC \right) \times h$$

$$= \frac{1}{3} \times \left(\frac{1}{2} \times 7 \times 4 \right) \times 6\ cm^3$$

$$= 28\ cm^3$$

Tutorial 3 Problem 2(b): Suggested Solution

2.(b) Calculate the volume of the pyramid. (Correct to 2 decimal places)



$PQ = 10\ cm$

$RQ = 6\ cm$

$h = 11\ cm$

$$RP = \sqrt{PQ^2 - RQ^2}$$
$$= \sqrt{100 - 36} = \sqrt{64} = 8$$

Volume of pyramid

$$= \frac{1}{3} \times \text{ area of base } \times \text{ height}$$
$$= \frac{1}{3} \times \left(\frac{1}{2} \times 8 \times 6 \right) \times 11\ cm^3$$
$$= 88\ cm^3$$

Volume of a cone

The same principle used to determine the volume of a pyramid can be used to determine the volume of a right cone

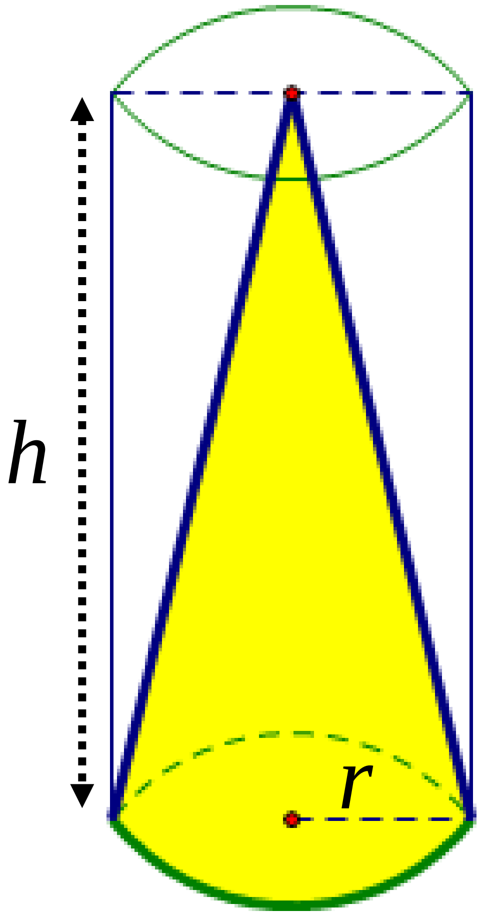
A right cone occupies one-third of the volume of the cylinder containing it.

$$\text{Volume of cone} = \frac{1}{3} \times \text{volume of cylinder}$$

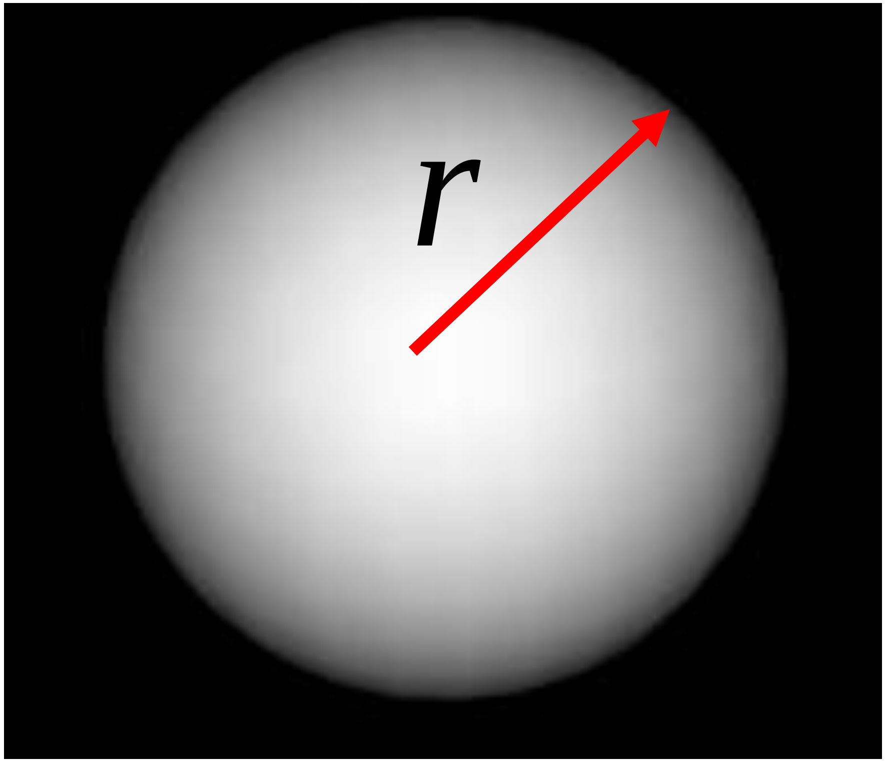
Volume of cone

$$= \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$\therefore V = \frac{1}{3}\pi r^2 h$$



Volume of a Sphere



Volume of a sphere with radius *r*

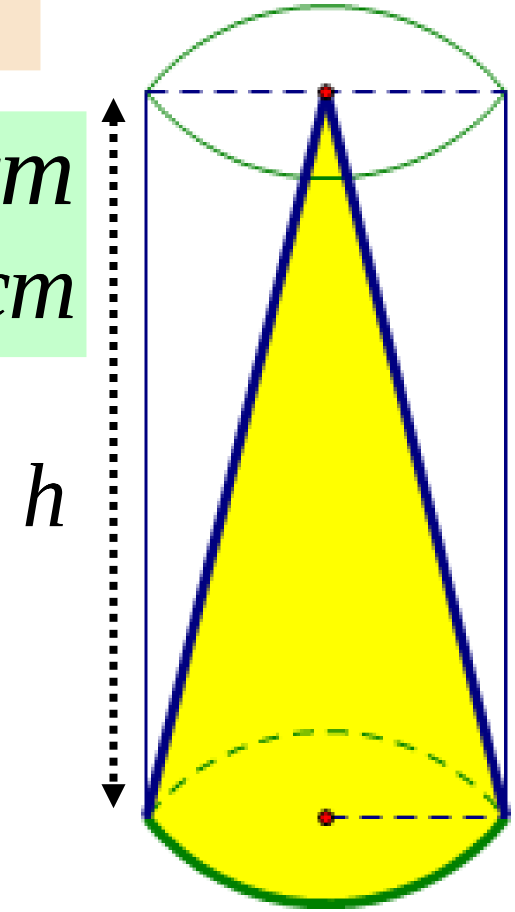
$$V = \frac{4}{3}\pi r^3$$

Calculate volume of Cone

Calculate the volume of the cone

(Correct to 2 decimal places):

$$r = 5,4\ cm$$
$$h = 10,2\ cm$$



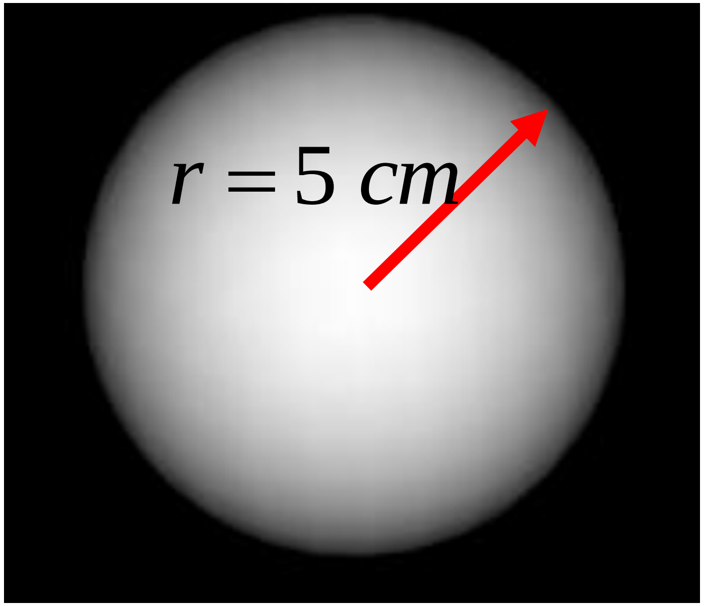
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (5,4)^2 (10,2)\ cm^3$$

$$= 311,47\ cm^3$$

Calculate volume of Sphere

Calculate the volume of the sphere (Correct to 2 decimal places):



$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi (5\ cm)^3$$

$$= 523{,}60\ cm^3$$

Tutorial 4: Volume of Cones and Spheres

1. Find the volumes of the cones with the following dimensions:

   (a) base radius 11,5 *mm* and height 23 *mm*,

   (b) base diameter 6,4 *cm* and height 8 *cm*.

2. Find the volume of a sphere of radius 15 *cm*.

PAUSE DVD

- Do Tutorial 4
- Then View solutions

Tutorial 4 Problem 1: Suggested Solutions

1. Find the volumes of the cones with dimensions:

   (a) base radius 11,5 *mm* and height 23 *mm*,

   (b) base diameter 6,4 *cm* and height 8 *cm*.

1\. (a) Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (11{,}5)^2 (23)\ mm^3$

$= 3185{,}31\ mm^3$

1\. (b) Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (3{,}2)^2 (8)\ cm^3$

$= 85{,}79\ cm^3$

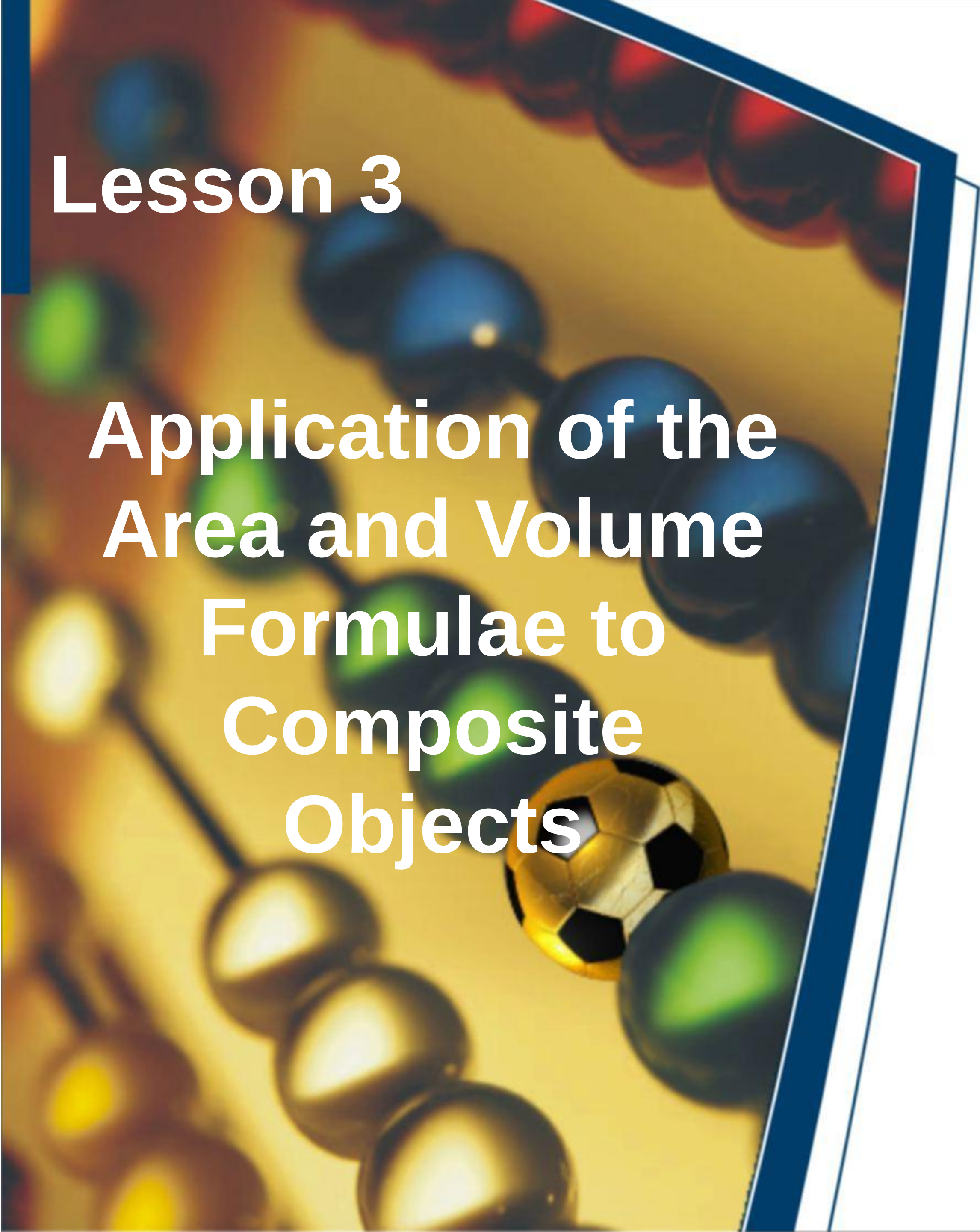
Tutorial 4 Problem 2: Suggested Solution

2. Find the volume of a sphere of radius 15 *cm*.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(15\ cm)^3$$

$$= 14\ 137{,}16\ cm^3$$



Nelson Mandela Metropolitan University

for tomorrow

Lesson 3

Application of the Area and Volume Formulae to Composite Objects

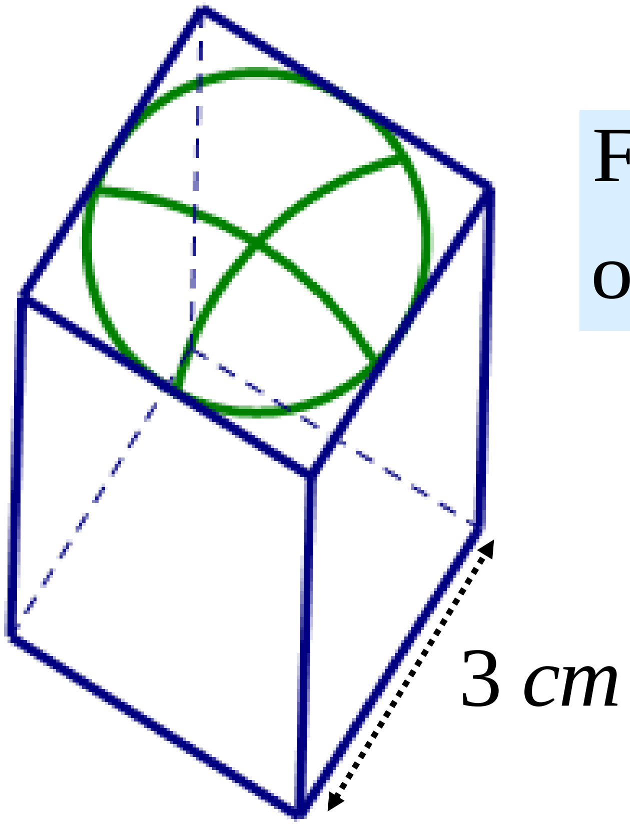
NCS Mathematics DVD Series

GOVAN MBEKI MATHEMATICS DEVELOPMENT UNIT

Empowering young minds

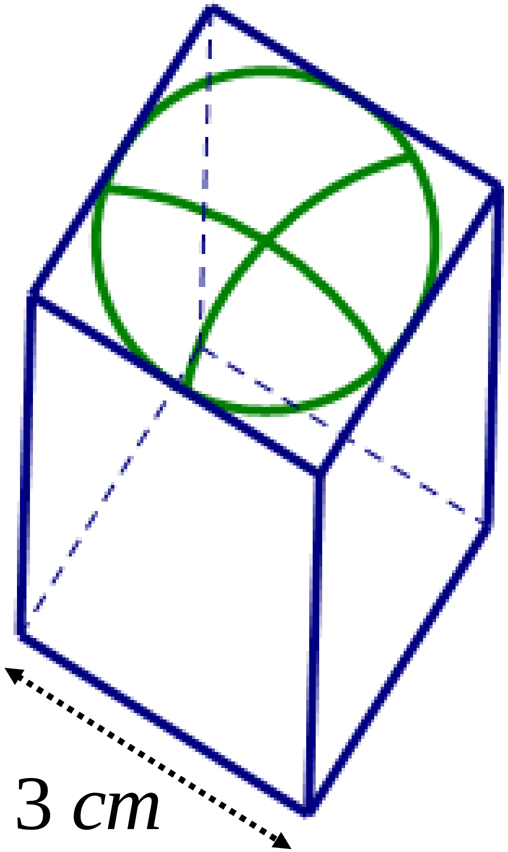
Combining a Cube and a Hemisphere

A hemisphere (half a sphere) is placed on top of a cube of side 3 *cm*.



Find the **total surface area** of the composite object.

Calculate total Surface Area



Total Surface Area

$$= 5s^2 + \frac{1}{2}\left(4\pi r^2\right) + \left(s^2 - \pi r^2\right)$$

$$= 5(3)^2 + 2\pi(1,5)^2 + (3)^2 - \pi(1,5)^2$$

$$= 54 + \pi(1,5)^2$$

$$= 61,07cm^2$$

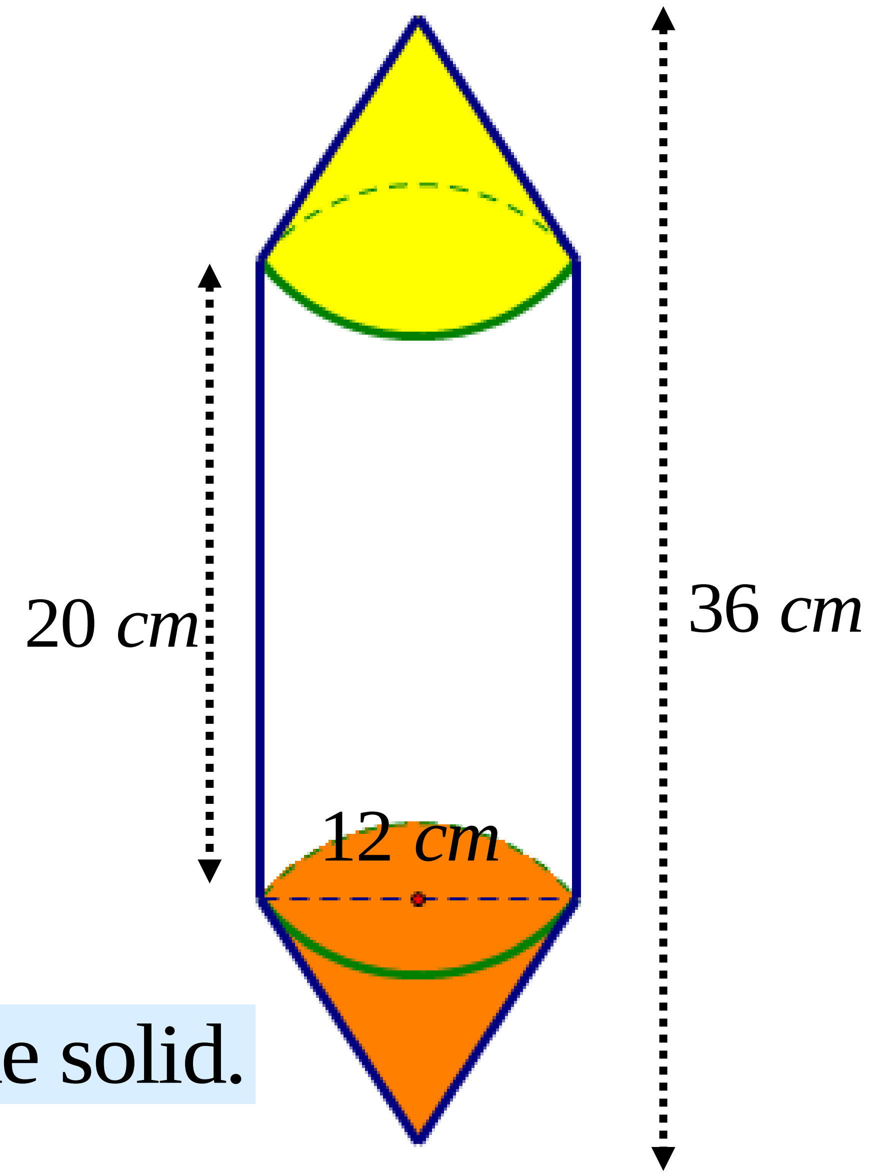
Combining a Cylinder and Cones

A solid metal object is formed by placing two identical cones at each end of a cylinder.

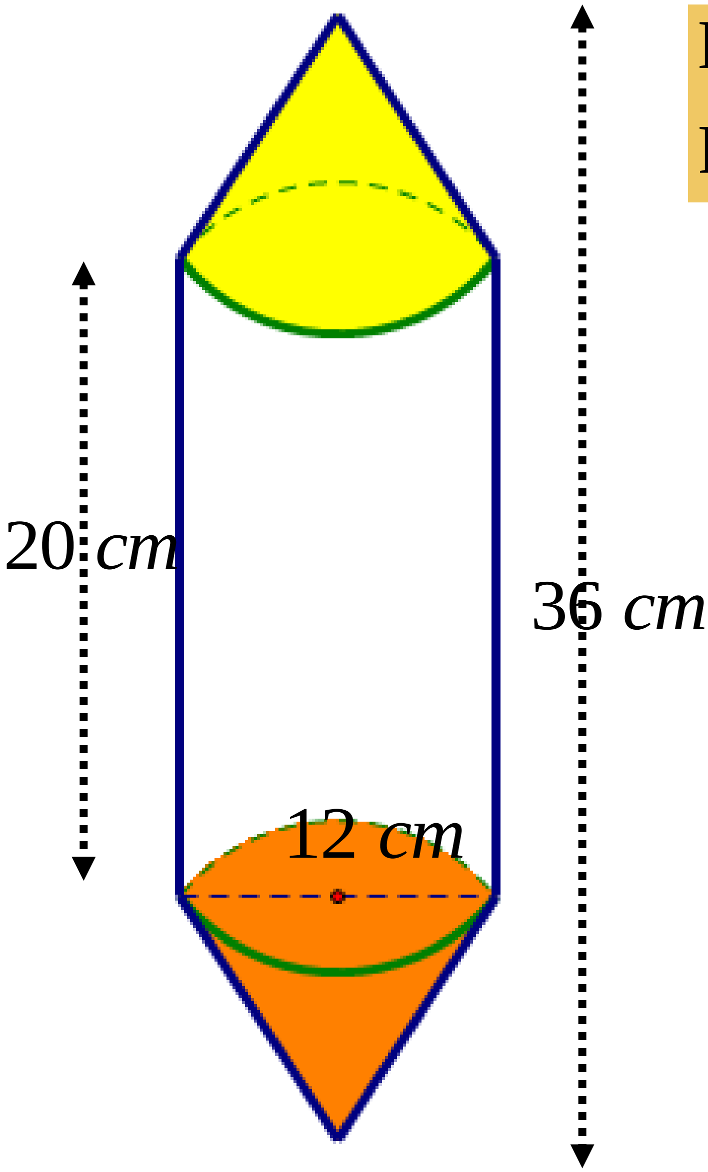
The total length of the object is 36 *cm*.

The cylinder is 20 cm long and has a diameter of 12 *cm*.

Calculate the volume of the solid.



Calculate the Total Volume



Height of cone $= 8\ cm$

Radius of cone (and cylinder) $= 6\ cm$

Volume of solid

$= $ Volume of cylinder + 2× volume of cone

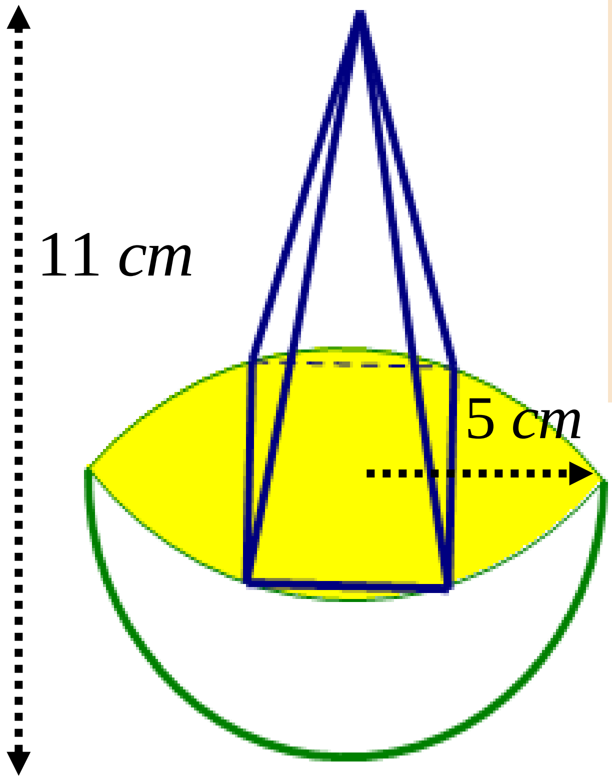
$$= \pi r^2 h + 2\left[\frac{1}{3} \times \pi r^2 h_1\right]$$

$$= \pi(6cm)^2(20cm) + \frac{2}{3}(\pi)(6cm)^2(8cm)$$

$$= 36\pi\left[20 + \frac{16}{3}\right] cm^3$$

$$= 2865{,}13\ cm^3$$

Tutorial 5: Surface Area and Volume of Composite Objects



The figure is an illustration of a modern artpiece which consists of a square-based pyramid sitting on the flat side of a hemisphere of radius 5 *cm*. The total height is 11 *cm*.

(1) Find the side length of the base of the pyramid.

(2) Find the height of the pyramid.

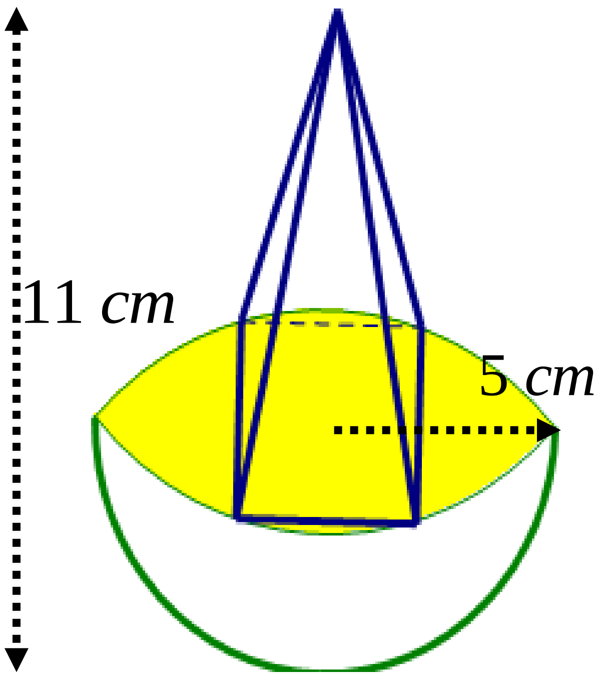
(3) Find the slanted height of the pyramid.

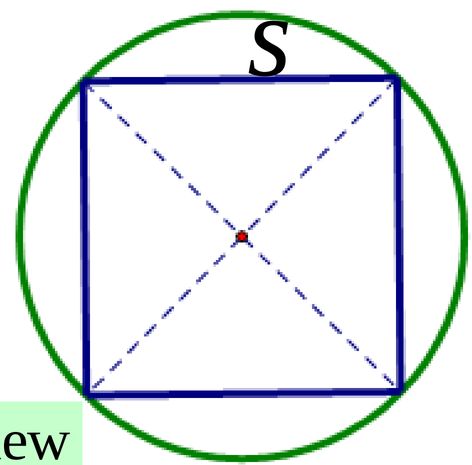
(4) Find the total surface area of the art piece.

PAUSE DVD

- Do Tutorial 5
- Then View Solutions

Tutorial 5 Problem 1: Suggested Solution





Top View

(1) Find the side length of the base of the pyramid.

Diagonal of square base

= the diameter of the sphere

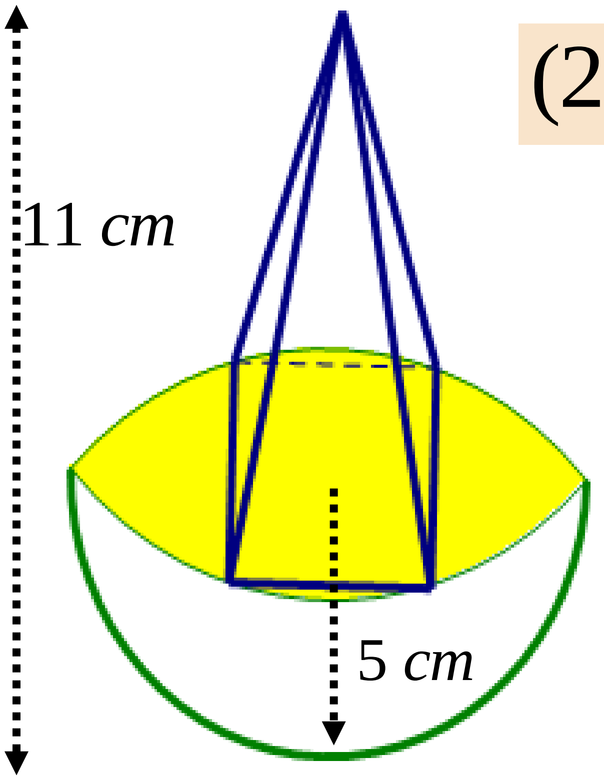
$\therefore$ Diagonal of square base = 10 cm

Hence, $s^2 + s^2 = 100$

So $2s^2 = 100$. Hence, $s^2 = 50$

Thus, $s = \sqrt{50} = 7{,}07 cm$

Tutorial 5 Problem 2: Suggested Solution



(2) Find the height of the pyramid.

Height of Pyramid

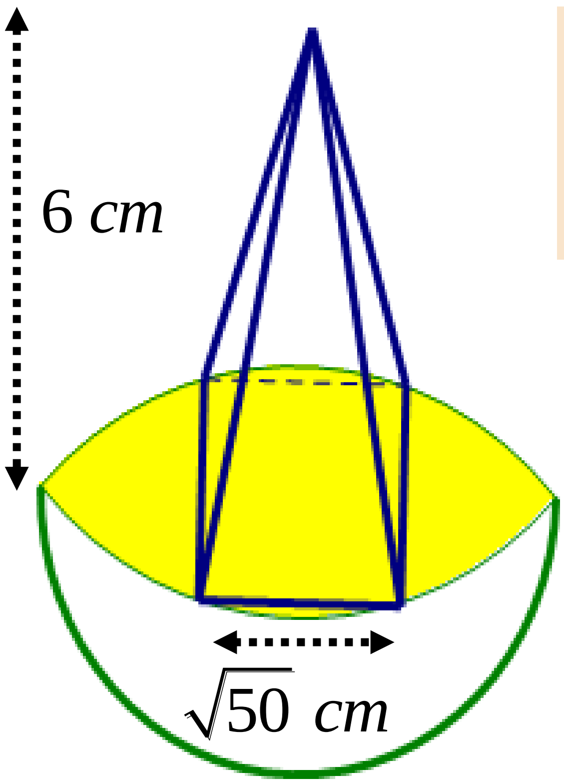
= Height of art piece

− Radius of hemisphere

$= 11\ cm - 5\ cm$

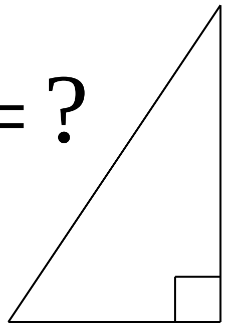
$= 6\ cm$

Tutorial 5 Problem 3: Suggested Solution



(3) Find the slanted height of the pyramid.

Consider the following triangle:

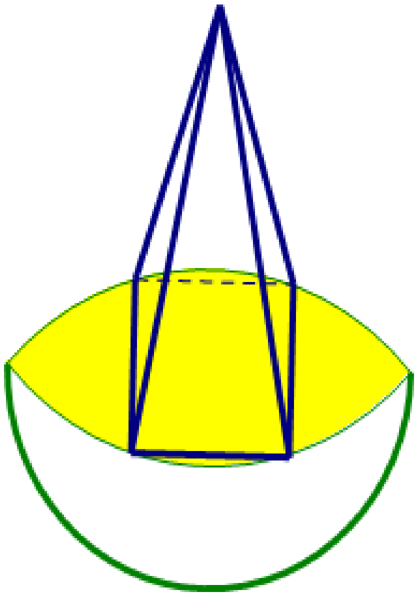


$$\frac{s}{2} = \frac{7,07}{2} = 3,53\ cm$$

$$l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2}$$

$$= \sqrt{6^2 + 3,53^2} = 6,96\ cm$$

Tutorial 5 problem 4: Suggested Solution



(4) Find the total surface area of the art piece.

Total surface area of art piece

$$4\times\frac{1}{2}sl+\frac{1}{2}\times4\pi r^2+\left(\pi r^2-s^2\right)$$

$$=\ 4\left(\frac{1}{2}(7,07)(6,96)\right)+\frac{1}{2}\left(4\pi(5)^2\right)+\pi(5)^2-(7,07)^2$$

$$=\left(98,41+235,61-49,98\right)\ cm^2$$

$$=284,04\ cm^2$$

Know that:

$r=5\ cm$

$s=7,07\ cm$

$l=6,96\ cm$

End of the DVD on Surface Area and Volume

<u>REMEMBER!</u>

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in the DVD.
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!