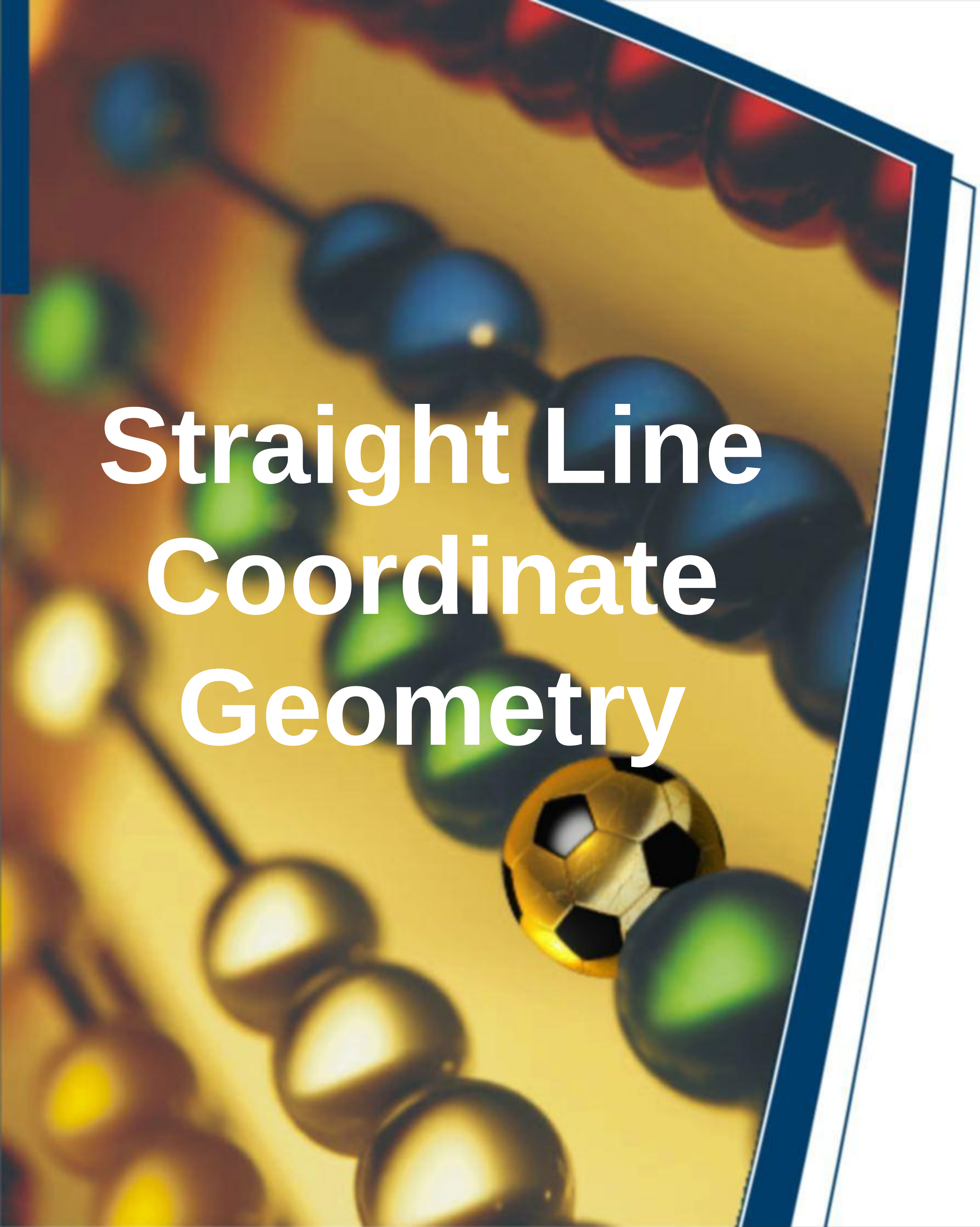

Straight Line Coordinate Geometry

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Outcomes for this DVD

In this DVD we will :

- Recall and apply distance, gradient and midpoint formulae.

LESSON 1

- Derive and apply a formula for the inclination of a line.

LESSON 2

- Derive and apply formulae for the equation of a line.

LESSON 3

Lesson 1

Distance, Gradient and Midpoint Formulae

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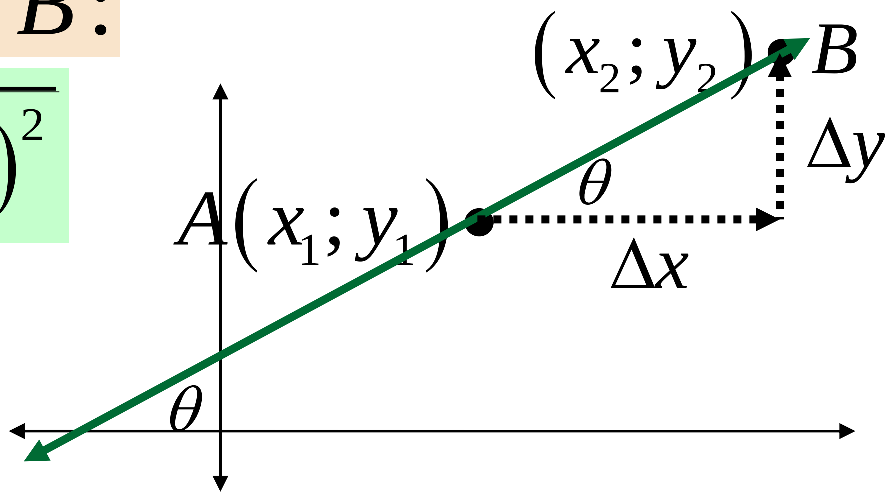
Distance, Gradient and Midpoint Formulae

You should know that for two points $A(x_1;y_1)$ and $B(x_2;y_2)$:

Distance between A and B:

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Gradient of line AB:

$$m_{\overleftrightarrow{AB}}=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\tan\theta$$

Co-ordinates of midpoint of segment AB:

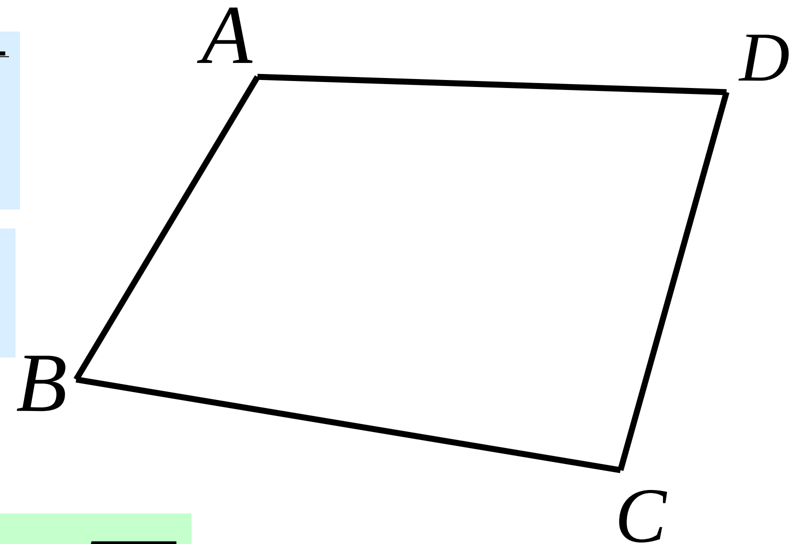
$$\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

Application of Distance Formula

$A(2;6),\ B(-5;2),\ C(4;-4)$ and $D(8;2)$ are the vertices of quadrilateral $ABCD$.

$$AB=\sqrt{(-5-2)^2+(6-2)^2}$$

$$=\sqrt{49+16}=\sqrt{65}$$



$$DC=\sqrt{4^2+6^2}=\sqrt{16+36}=\sqrt{52}$$

$\therefore$ $ABCD$ is not a parallelogram nor a rectangle.

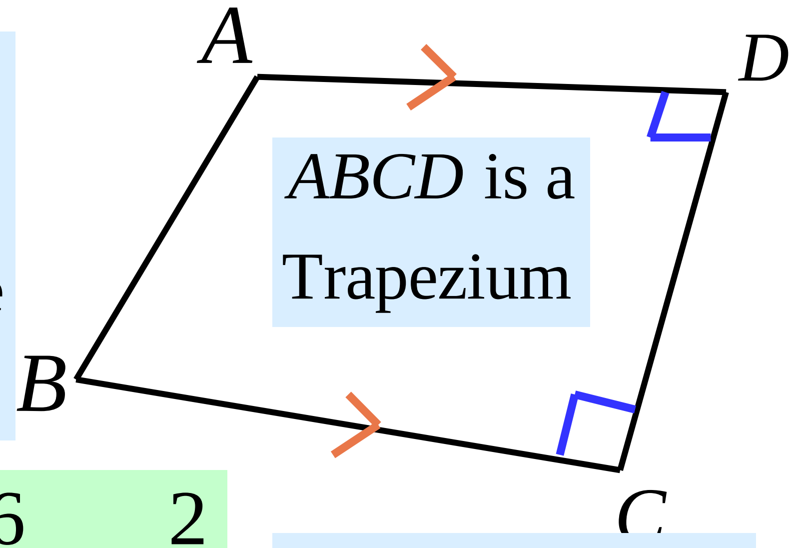
Application of Gradient Formula

$A(2;6)$, $B(-5;2)$, $C(4;-4)$ and $D(8;2)$ are the vertices of quadrilateral $ABCD$.

Know: $AB \neq CD$

$$m_{\overline{AB}} = \frac{2-6}{-5-2} = \frac{-4}{-7} = \frac{4}{7}$$

$\therefore$ Angle between $\overleftrightarrow{AB}$ and the positive $x-$axis is acute.



$ABCD$ is a Trapezium

$$m_{\overline{AD}} = \frac{4}{-6} = -\frac{2}{3} \text{ and } m_{\overline{BC}} = \frac{-6}{9} = -\frac{2}{3}$$

$$\therefore \overline{AD} \mathbin{//} \overline{BC}$$

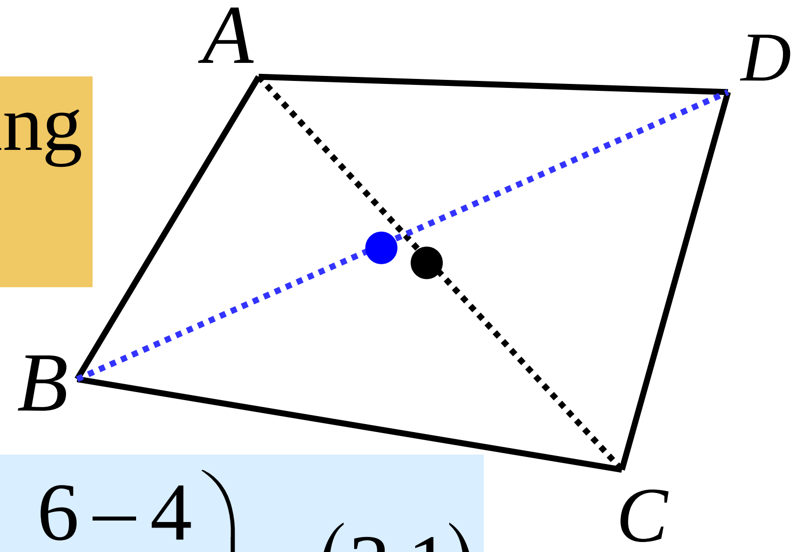
$$m_{\overline{CD}} = \frac{-6}{-4} = \frac{3}{2}$$

$$\therefore m_{\overline{AD}} \times m_{\overline{CD}} = -1 \text{ and } m_{\overline{BC}} \times m_{\overline{CD}} = -1$$

$$\therefore \overline{CD} \perp \overline{AD} \text{ and } \overline{CD} \perp \overline{BC}$$

Application of Midpoint Formula

$A(2;6)$, $B(-5;2)$, $C(4;-4)$ and $D(8;2)$ are the vertices of quadrilateral $ABCD$.



$\therefore$ Diagonals are not bisecting each other.

Midpoint of $\overline{AC}$ is $\left(\dfrac{2+4}{2};\dfrac{6-4}{2}\right)=(3;1)$

Midpoint of $\overline{BD}$ is $\left(\dfrac{-5+8}{2};\dfrac{2+2}{2}\right)=\left(\dfrac{3}{2};2\right)$

Tutorial 1: Distance, Gradient and Midpoint Formulae

Given a quadrilateral with vertices as indicated.

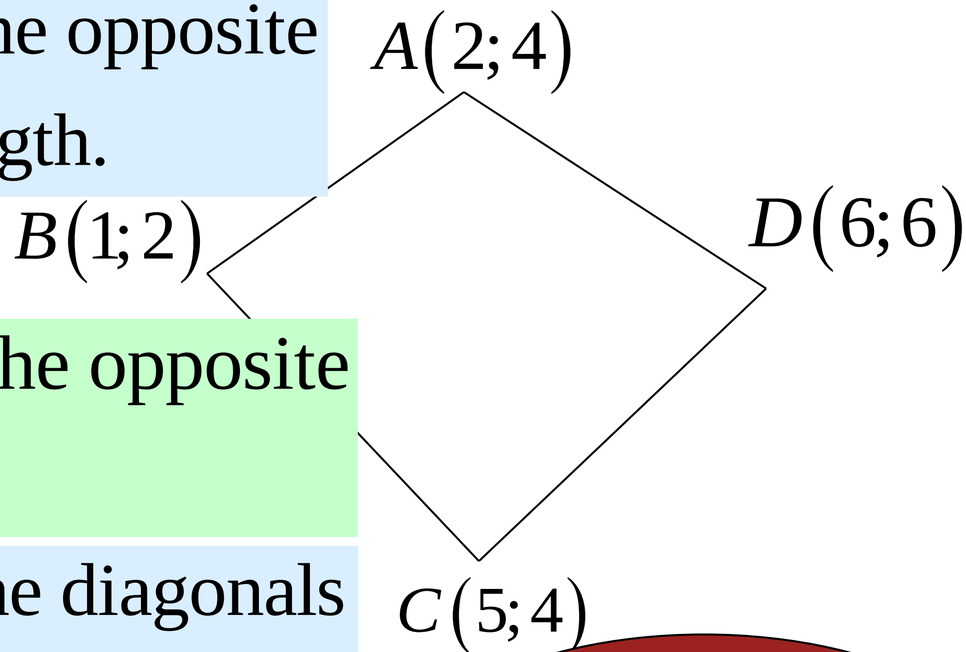
1) Determine whether the opposite sides are equal in length.

2) Determine whether the opposite sides are parallel.

3) Determine whether the diagonals bisect each other.

4) Classify this quadrilateral.

5) Can this quadrilateral be classified as a rectangle?

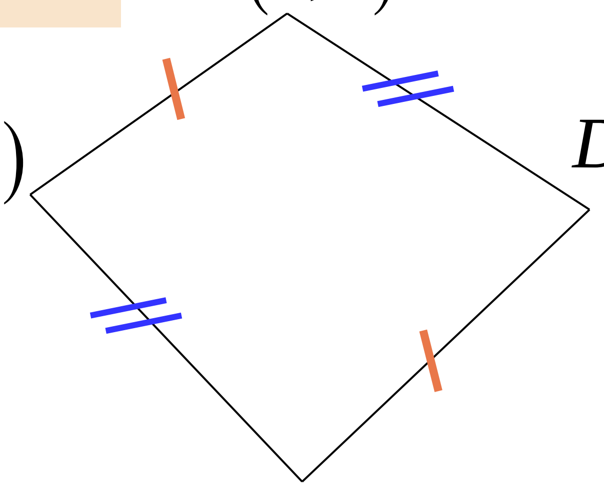


PAUSE DVD

- Do Tutorial 1
- Then View Solutions

Tutorial 1 Problem 1: Suggested Solution

1) Determine whether the opposite sides are equal in length.



$AB = \sqrt{1+4} = \sqrt{5}$

and $CD = \sqrt{1+4} = \sqrt{5}$

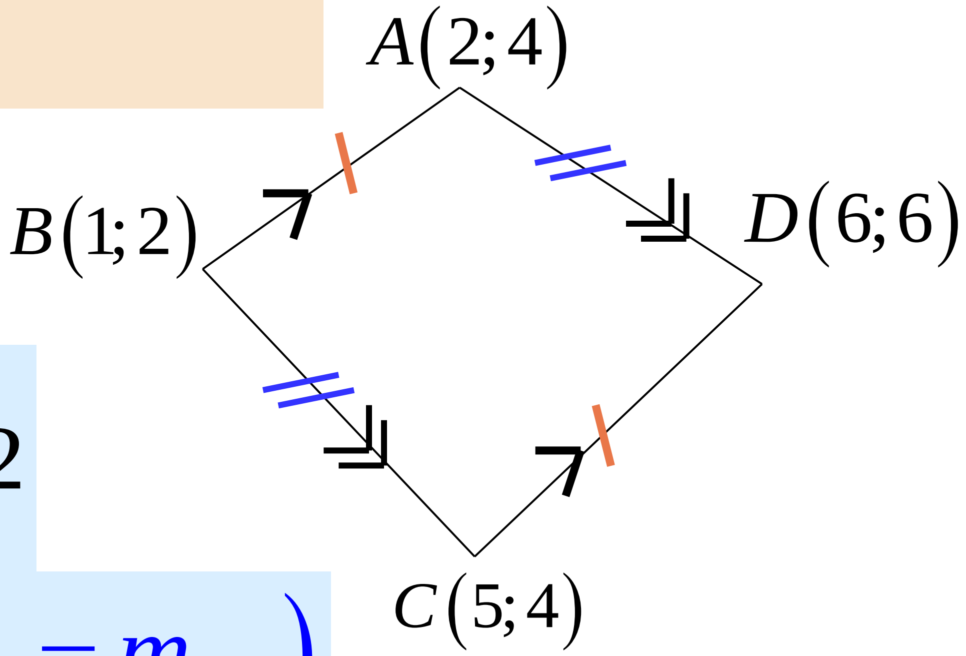
$\therefore AB = CD$

Similarily $AD = \sqrt{16+4} = \sqrt{20} = BC$

$\therefore$ Opposite sides are equal.

Tutorial 1 Problem 2: Suggested Solution

2) Determine whether the opposite sides are parallel.



$$m_{\overline{AB}} = \frac{4-2}{2-1} = 2$$

and $$m_{\overline{CD}} = \frac{6-4}{6-5} = 2$$

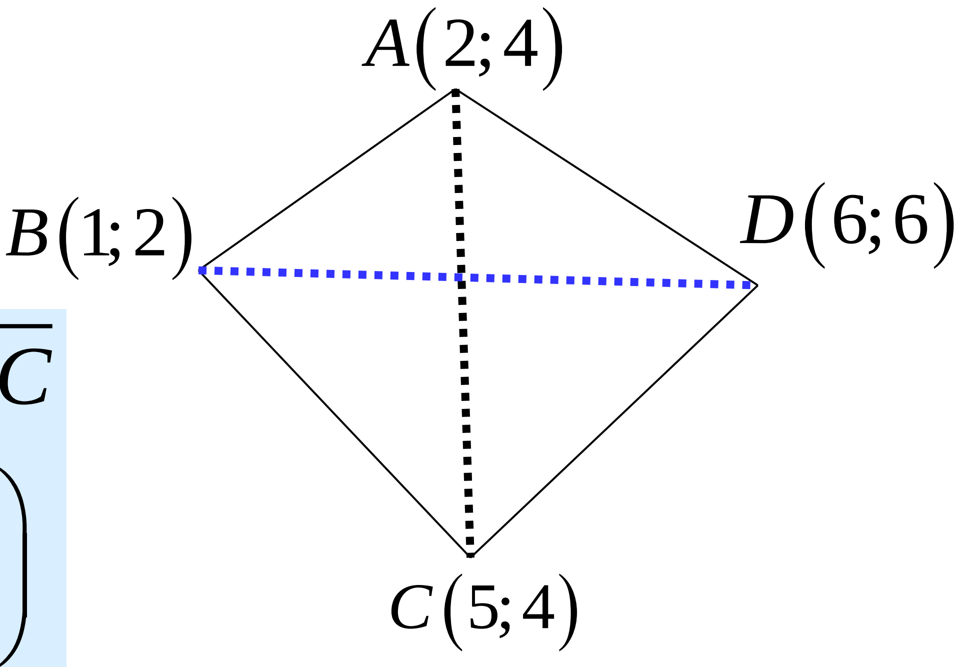
$$\therefore \overline{AB} // \overline{CD} \quad \left(\because m_{\overline{AB}} = m_{\overline{CD}}\right)$$

$$m_{\overline{AD}} = m_{\overline{BC}} = \frac{2}{4} = \frac{1}{2}$$

$\Rightarrow \overline{AD} // \overline{BC}$ ($\because$ Gradients are equal)

Tutorial 1 Problem 3: Suggested Solution

3) Determine whether the diagonals bisect each other.



Midpoint of diagonal $\overline{AC}$

is $\left(\frac{2+5}{2};\frac{4+4}{2}\right)=\left(\frac{7}{2};4\right)$

Midpoint of diagonal $\overline{BD}$

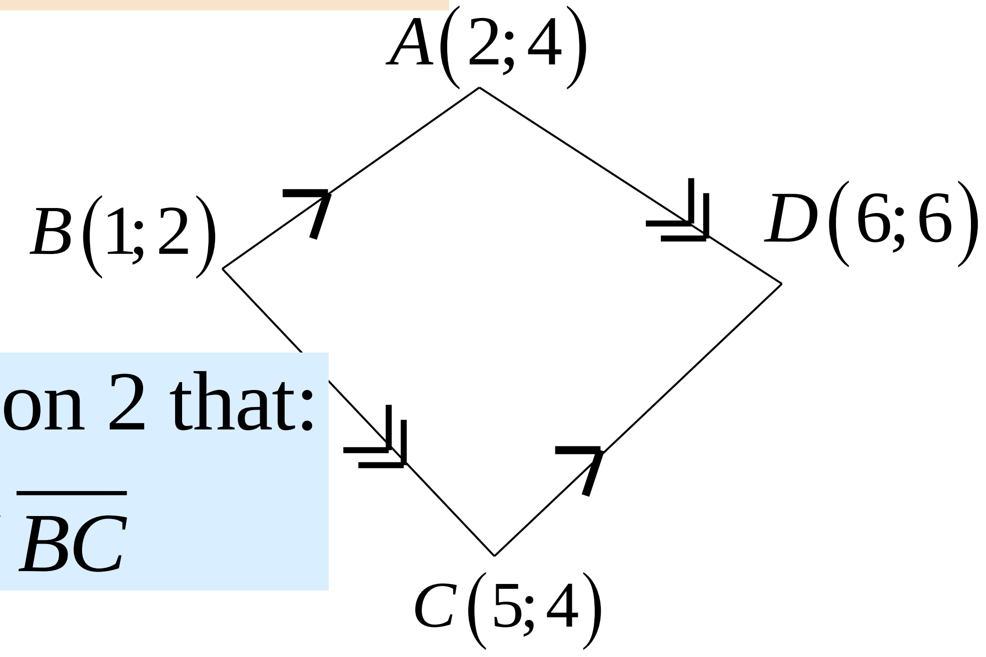
is $\left(\frac{1+6}{2};\frac{2+6}{2}\right)=\left(\frac{7}{2};4\right)$

$\therefore$ Diagonals bisect each other

at the point $\left(\frac{7}{2};4\right)$

Tutorial 1 Problem 4: Suggested Solution

4) Classify this quadrilateral.



We showed in question 2 that:

$\overline{AB} // \overline{DC}$ and $\overline{AD} // \overline{BC}$

$\therefore ABCD$ can be classified as a parallelogram.

Tutorial 1 Problem 5: Suggested Solution

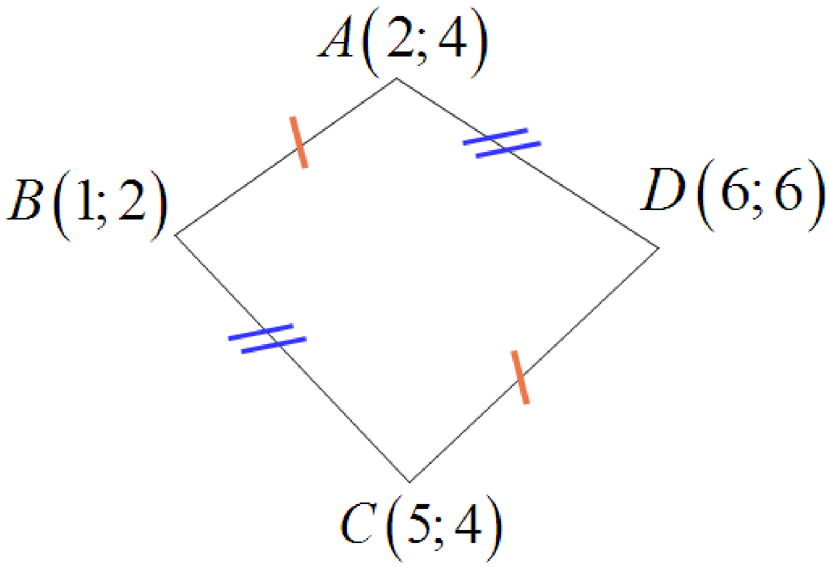
5) Can this quadrilateral be classified as a rectangle?

In question 1 we showed that opposite sides are equal.

We showed in question 2 that:

$$m_{\overline{AB}} = 2 \text{ and } m_{\overline{BC}} = \frac{1}{2}$$

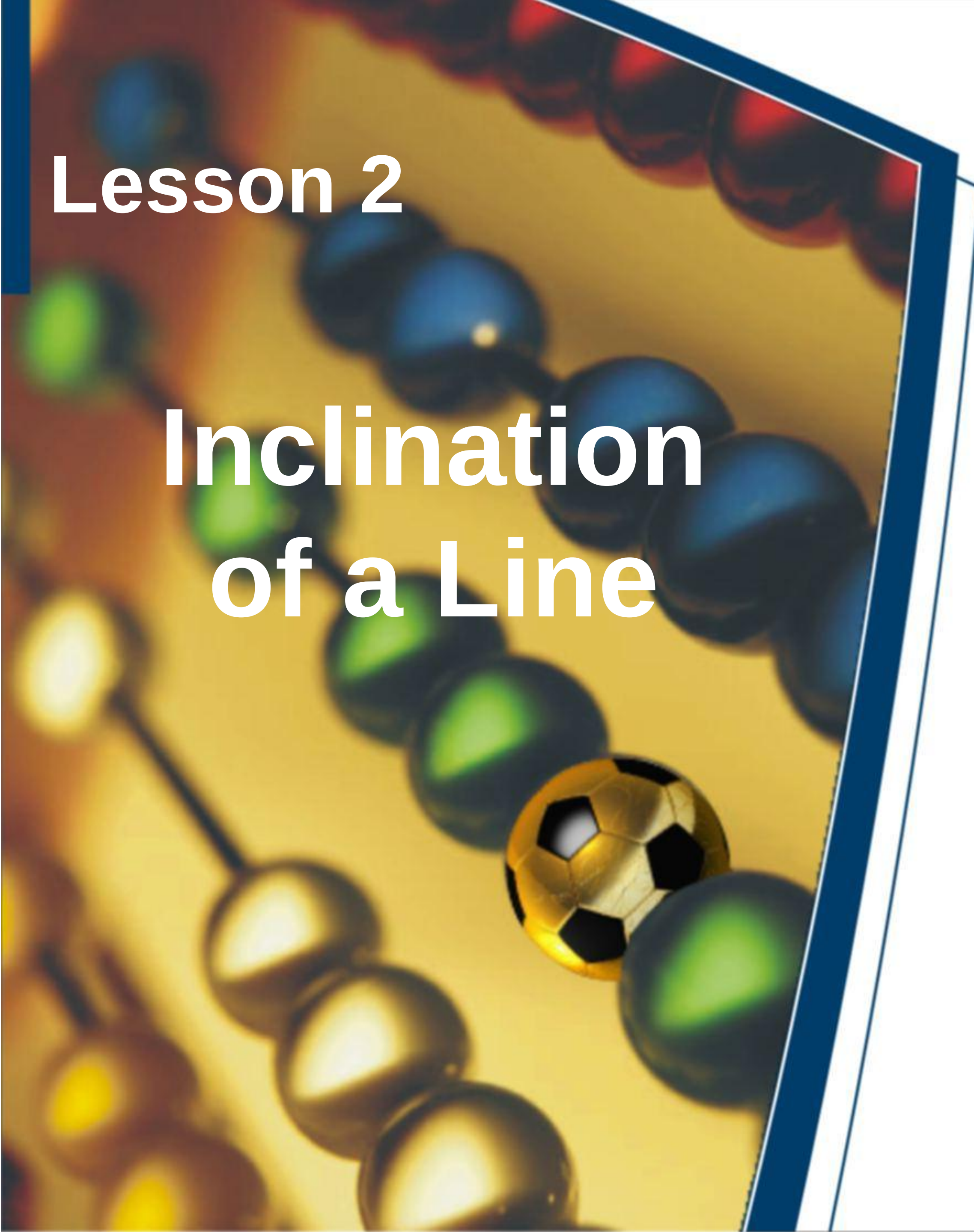
$$\therefore m_{\overline{AB}} \times m_{\overline{BC}} \neq -1$$



$\therefore \angle B \neq 90^\circ \Rightarrow ABCD$ is not a rectangle.

Lesson 2

Inclination of a Line



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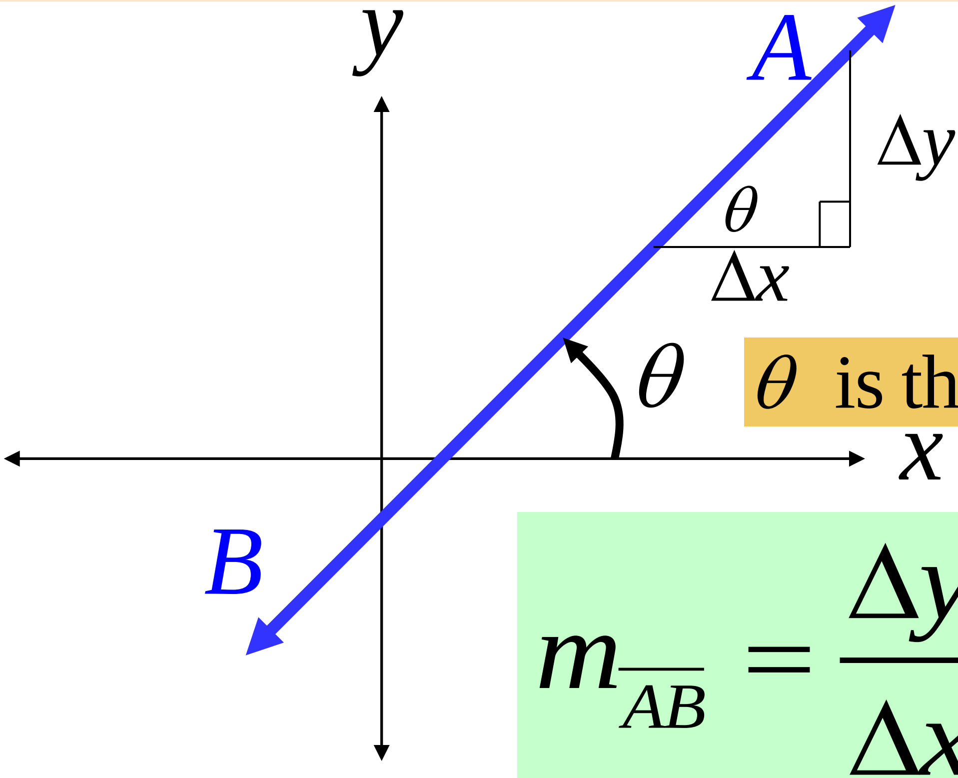
What is the Inclination of a Line?

The angle that a line makes with the positive direction of the $x-$axis is called the inclination of the line.



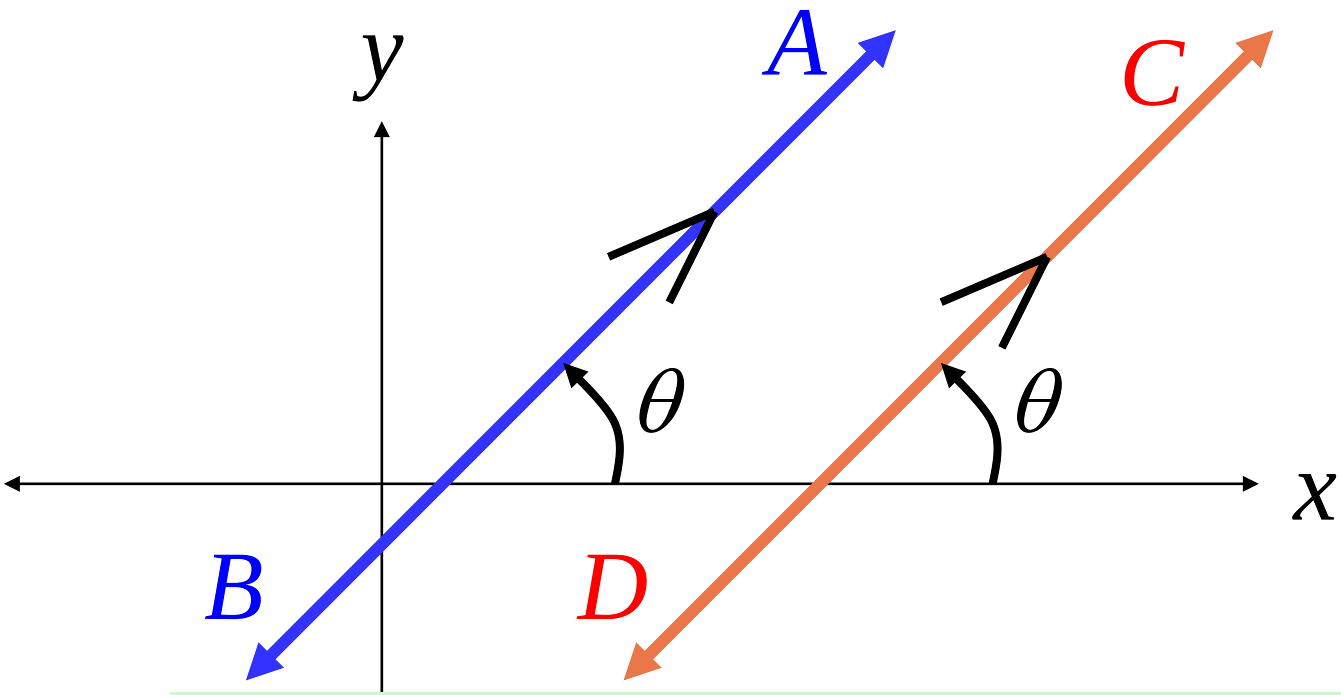
θ is the Inclination of the line

$$m_{\overline{AB}} = \frac{\Delta y}{\Delta x} = \tan\theta$$

$$\text{for } 0° \le \theta \le 180°$$

When are two Lines Parallel?

Two lines with the same inclination will be parallel.



$$\overleftrightarrow{AB} // \overleftrightarrow{CD} \Leftrightarrow m_{\overleftrightarrow{AB}} = m_{\overleftrightarrow{CD}} = \tan\theta$$

When are two Lines Perpendicular?

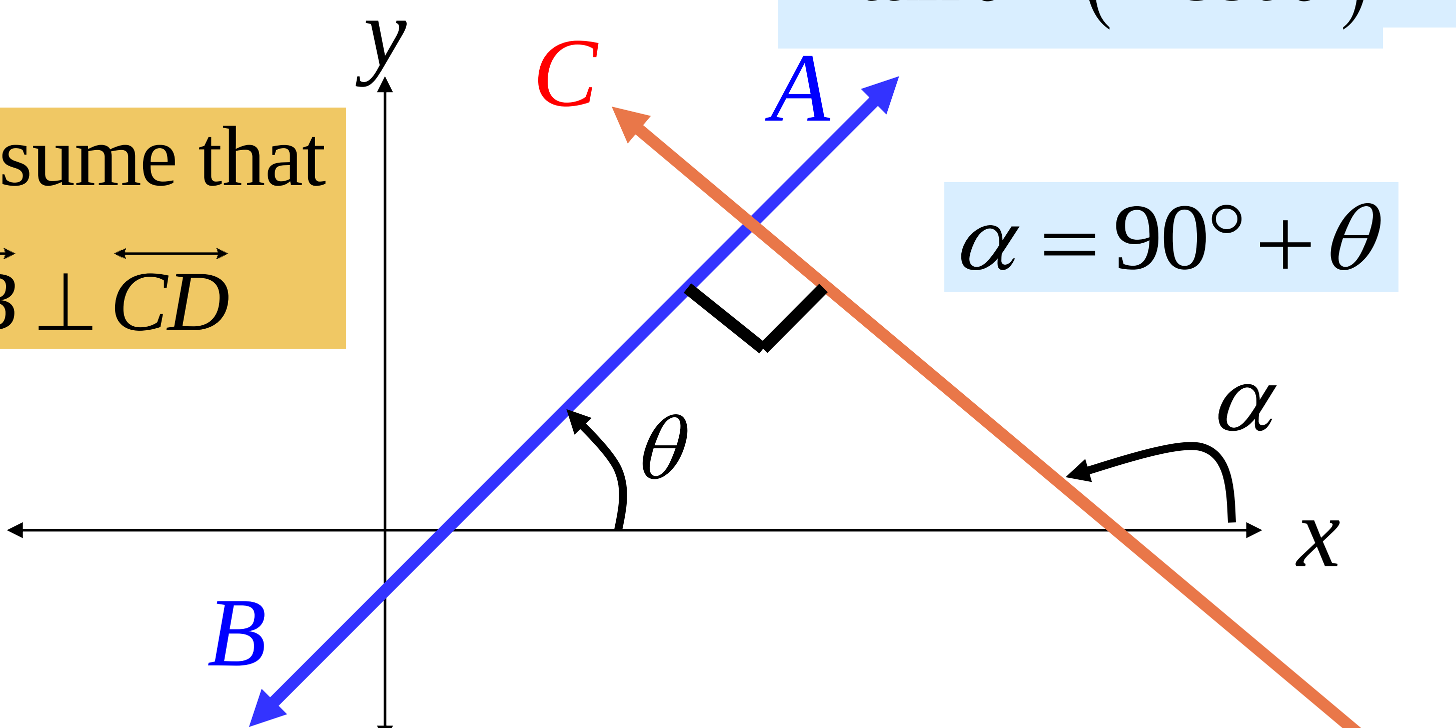
$$m_{AB} = \tan\theta \quad \text{and} \quad m_{CD} = \tan\alpha$$

$$\therefore m_{AB} \times m_{CD} = \tan\theta \times \tan\alpha = \tan\theta \times \tan\left(90^\circ + \theta\right)$$

$$= \tan\theta \times \left(-\cot\theta\right) = -1$$

Assume that

$$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$$



$$\alpha = 90^\circ + \theta$$

$$\therefore \quad \overleftrightarrow{AB} \perp \overleftrightarrow{CD} \Leftrightarrow m_{\overleftrightarrow{AB}} \times m_{\overleftrightarrow{CD}} = -1$$

Transforming Inclination into Gradient

Example : Given that the inclination of a line is 78°, find the gradient of the line.

$$m_{\text{Line}} = \tan 78° = 4,705$$

Example : Given that the inclination of a line is 158°, find the gradient of the line.

$$m_{\text{Line}} = \tan 158° = \tan\left(180° - 22°\right)$$

$$= -\tan 22° = -0,404$$

Transforming Gradient to Inclination

Example: Find the inclination of a line if the the gradient of the line is $\frac{3}{4}$.

$$m = \tan\theta \Rightarrow \tan\theta = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\frac{3}{4} \simeq 36,87^\circ$$

Example: Find the inclination of a line if the the gradient of the line is $-\frac{5}{6}$.

$$\tan\theta = -\frac{5}{6} \Rightarrow \textbf{Reference Angle} = \tan^{-1}\frac{5}{6} \simeq 39,81^\circ$$

$$\Rightarrow \theta = 180^\circ - 39,81^\circ = 140,19^\circ$$

Tutorial 2: Inclination of a Line

1) Given that $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and that $m_{\overleftrightarrow{AB}} = 1,7$.

Determine the inclination of line CD.

2) The inclination of line AB is $73°$.

Determine the inclination of line

CD if $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.

PAUSE DVD

- Do Tutorial 2
- Then View Solutions

Tutorial 2 Problem 1: Suggested Solution

1) Given that $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and that $m_{\overleftrightarrow{AB}} = 1,7$.

Determine the inclination of line CD.

$$m_{\overline{CD}} = 1,7$$

$$\therefore \tan\theta = 1,7$$

$$\therefore \theta = \tan^{-1} 1,7 \Rightarrow \theta = 59,5^\circ$$

$$\therefore \text{ Inclination of } \overleftrightarrow{CD} \text{ is } 59,5^\circ$$

Tutorial 2 Problem 2: Suggested Solution

2) The inclination of line AB is 73°.

Determine the inclination of line CD if

$\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.

$$\therefore m_{\overline{CD}} = -\frac{1}{m_{\overline{AB}}} \quad \text{and } m_{\overline{AB}} = \tan 73^\circ$$

$$\Rightarrow m_{\overline{CD}} = -\frac{1}{\tan 73^\circ}$$

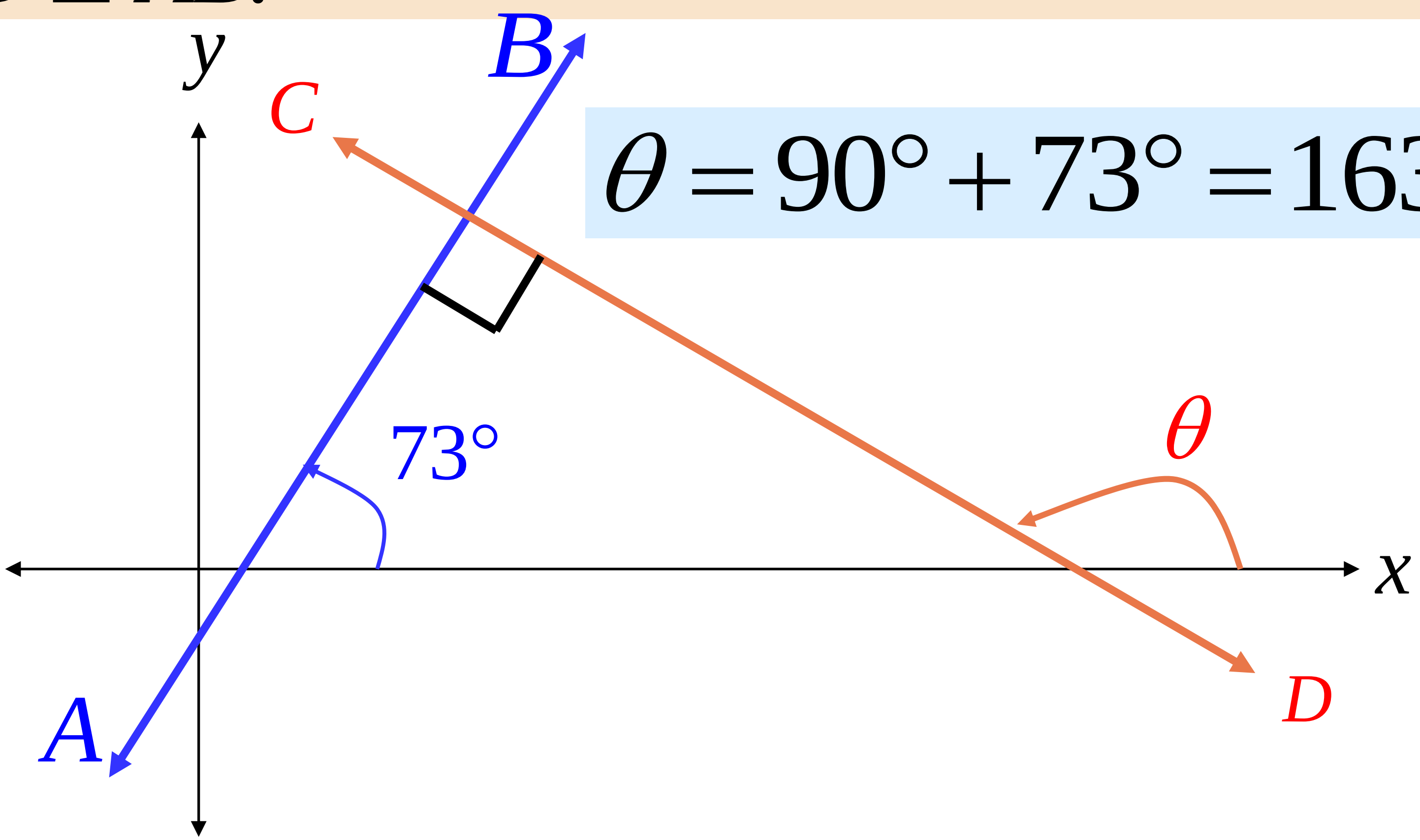
$$\therefore \text{ Inclination of } \overleftrightarrow{CD} \text{ is } 180^\circ - \tan^{-1}\left(\frac{1}{\tan 73^\circ}\right)$$

$$\text{or } 180^\circ - 17^\circ \text{ or } 163^\circ$$

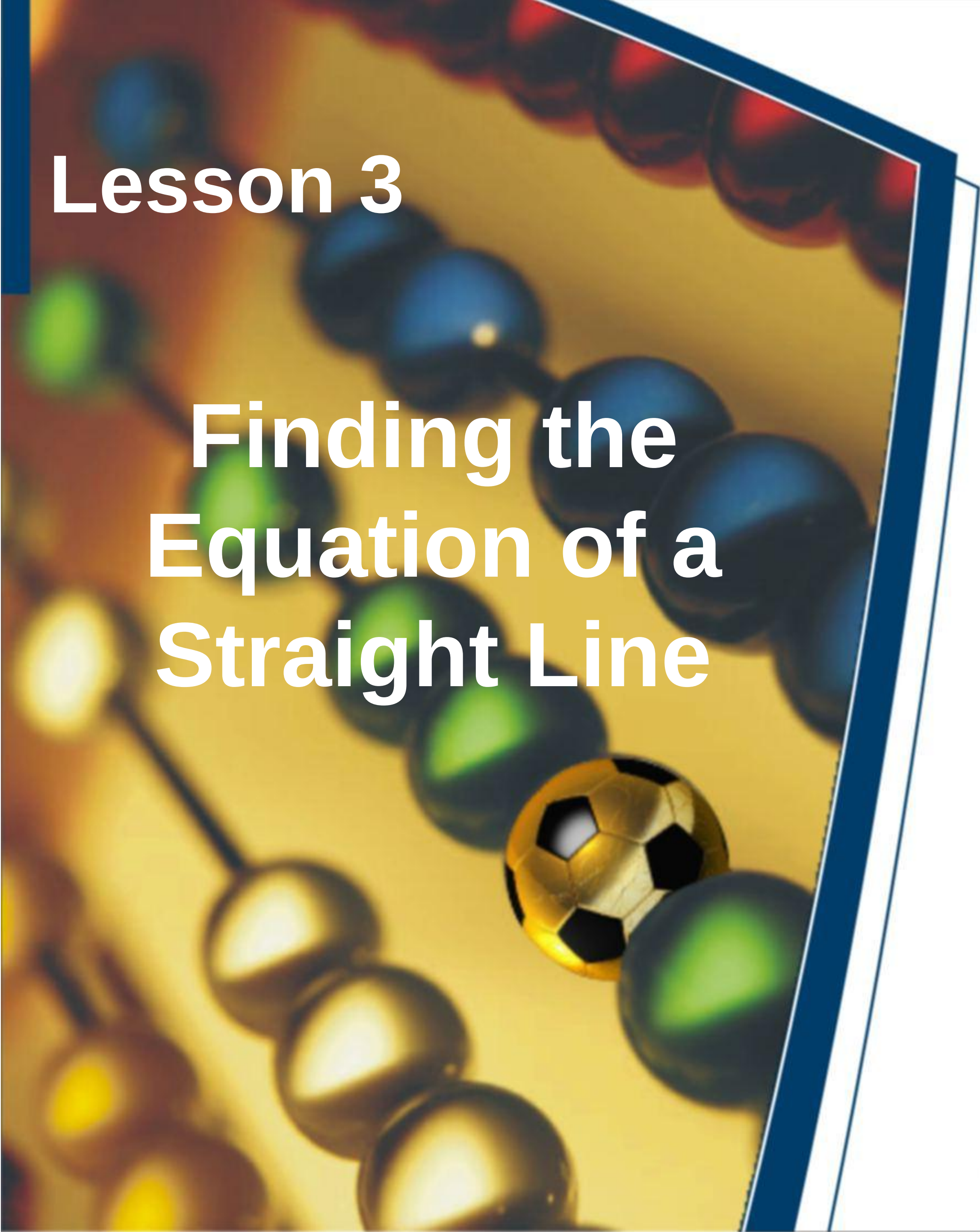
Tutorial 2 Problem 2: Alternative Solution

2) The inclination of line AB is 73°.

Determine the inclination of line CD if $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.



$$\theta = 90° + 73° = 163°$$



Lesson 3

Finding the Equation of a Straight Line

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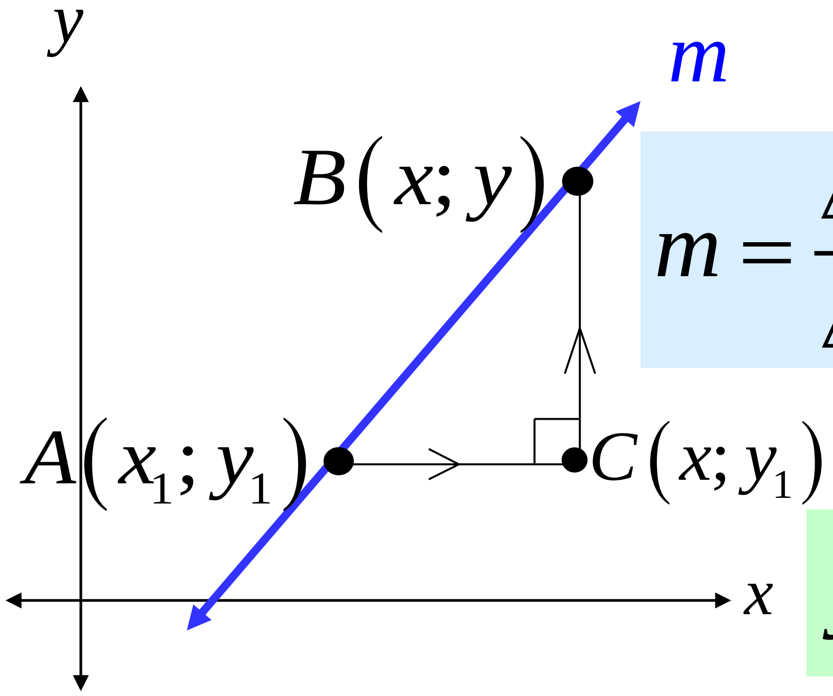
Point – Gradient Form of a Straight Line

Let $A(x_1; y_1)$ be a fixed point on a line with gradient m.

Let $B(x; y)$ be any point on the line.



$$m = \frac{\Delta y}{\Delta x} = \frac{CB}{AC} = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

Point – Gradient form of straight line

Utilize the Point-Gradient Formula

Find the equation of the line passing through the point $(3;-5)$ that has a gradient of $-\frac{2}{3}$.

Use the point-gradient formula.

$$y - y_1 = m(x - x_1)$$

$\therefore$ Equation of line is given by $y + 5 = -\frac{2}{3}(x - 3)$

or $y = -\frac{2x}{3} + (2 - 5)$ or $y = -\frac{2x}{3} - 3$

Standard form of a straight line.

2 – Point Form of a Straight line

Let $A(x_1; y_1)$ and $B(x_2; y_2)$ be two fixed points on a line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$ is the gradient of this line.

Use point – gradient form of line:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Two – Point form of straight line

Utilize the two - point formula

Find the equation of the line passing through the points $(-3;-5)$ and $(4;3)$.

Use the two – point formula:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Alternatively:

$$y + 5 = \frac{8}{7}(x + 3)$$

$\therefore$ Equation of line is given by $y - 3 = \dfrac{-5-3}{-3-4}(x-4)$

or $y - 3 = \dfrac{8}{7}(x - 4)$ or $y = \dfrac{8x}{7} - \dfrac{11}{7}$

Tutorial 3: Part 1: Equations of a Straight Line

1) Find the equation of the line that:

(a) Is parallel to the line $y = -3x - 2$ and passes through $(3;-4)$.

(b) Is perpendicular to the line $y + \frac{2x}{3} - 3 = 0$ and passes through the point $(-4;5)$.

PAUSE DVD

- Do Tutorial 3 (Part 1)
- Then View Solutions

Tutorial 3 Problem 1(a): Suggested Solution

(1a) Find the equation of the line that is parallel to the line $y = -3x - 2$ and passes through $(3; -4)$.

$m = -3$ is the gradient of the line.

Use the point – gradient formula:

$$y - y_1 = m(x - x_1)$$

$\therefore$ Equation of line is given by $y + 4 = -3(x - 3)$

or $y = -3x + 5$

Tutorial 3 Problem 1(b): Suggested Solution

(1b) Find the equation of the line that is perpendicular to the line $y+\frac{2x}{3}-3=0$ and passes through the point $(-4;5)$.

$$y=-\frac{2x}{3}+3 \Rightarrow \text{ Gradient of given line is } -\frac{2}{3}$$

$$\therefore \text{ Gradient of line perpendicular to given line is } \frac{3}{2}$$

From point – gradient formula the equation of required line is given by:

$$y-5=\frac{3}{2}(x+4) \text{ or } y=\frac{3x}{2}+11$$

Tutorial 3: Part 2: Equations of a Straight Line

2) $P(-2;-4)$, $Q(-4;2)$ and $R(7;1)$ are the vertices of ΔPQR and M is the midpoint of $\overline{PQ}$.

(a) Find the equation of line MR.

(b) Is $\overline{MR} \perp \overline{PQ}$?

(c) Determine the equation of the line through M that is perpendicular to $\overline{PQ}$.

(d) Find the equation of the line that is parallel to $\overline{PR}$ and passes through M.

PAUSE DVD

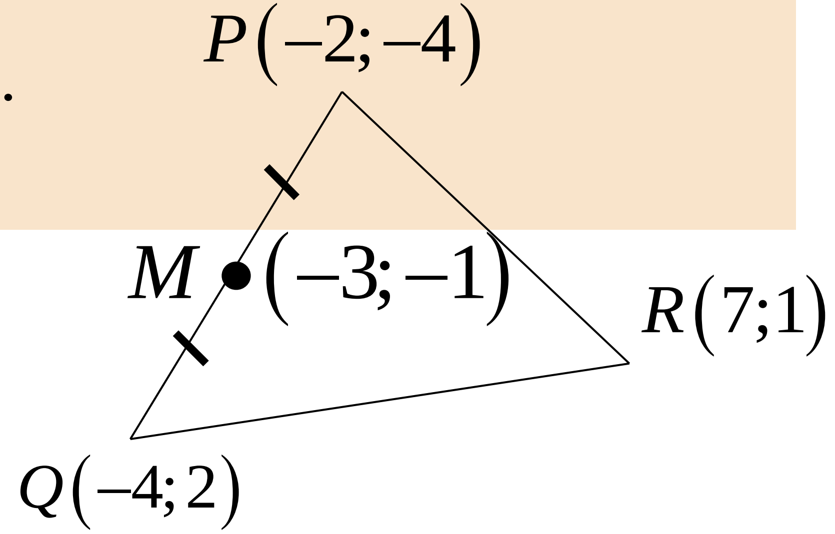
- Do Tutorial 3 (Part 2)
- Then View Solutions

Tutorial 3 Problems 2 (a) and (b): Suggested Solutions

2) $P(-2;-4)$, $Q(-4;2)$ and $R(7;1)$ are the vertices of ΔPQR and M is the midpoint of $\overline{PQ}$.

(a) Find the equation of line MR.

(b) Is $\overline{MR} \perp \overline{PQ}$?



2(a) $M(-3;-1)$

$$m_{\overline{MR}} = \frac{-2}{-10} = \frac{1}{5}$$

Equation of $\overleftrightarrow{MR}$: $y-1=\frac{1}{5}(x-7)$ or $y=\frac{x}{5}-\frac{2}{5}$

2(b) $m_{\overline{PQ}} = \frac{-6}{2} = -3 \Rightarrow \overline{MR} \not\perp \overline{PQ}$ (Reason?)

Tutorial 3 Problem 2 (c): Suggested Solutions

2) $P(-2;-4)$, $Q(-4;2)$ and $R(7;1)$ are the vertices of ΔPQR and M is the midpoint of $\overline{PQ}$.

(c) Determine the equation of the line through M that is perpendicular to $\overleftrightarrow{PQ}$.

2(c) $m_{\overline{PQ}} = -3$ $\therefore$ Gradient of line $\perp$ to $\overline{PQ} = \frac{1}{3}$

Equation of requested line:

$y+1=\frac{1}{3}(x+3)$ or $y=\frac{x}{3}$ Know: $M(-3;-1)$

Tutorial 3 Problem 2 (d): Suggested Solutions

2) $P(-2;-4)$, $Q(-4;2)$ and $R(7;1)$ are the vertices of ΔPQR and M is the midpoint of $\overline{PQ}$.

Know: $M(-3;-1)$

(d) Find the equation of the line that is parallel to $\overline{PR}$ and passes through M.

2(d)

$$m_{\text{LINE}} = m_{\overline{PR}} = \frac{-5}{-9} = \frac{5}{9}$$

Equation:

$$y+1=\frac{5}{9}(x+3) \text{ or } y=\frac{5x}{9}+\frac{2}{3}$$

End of the DVD on Straight Line Coordinate Geometry

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in the DVD.
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!