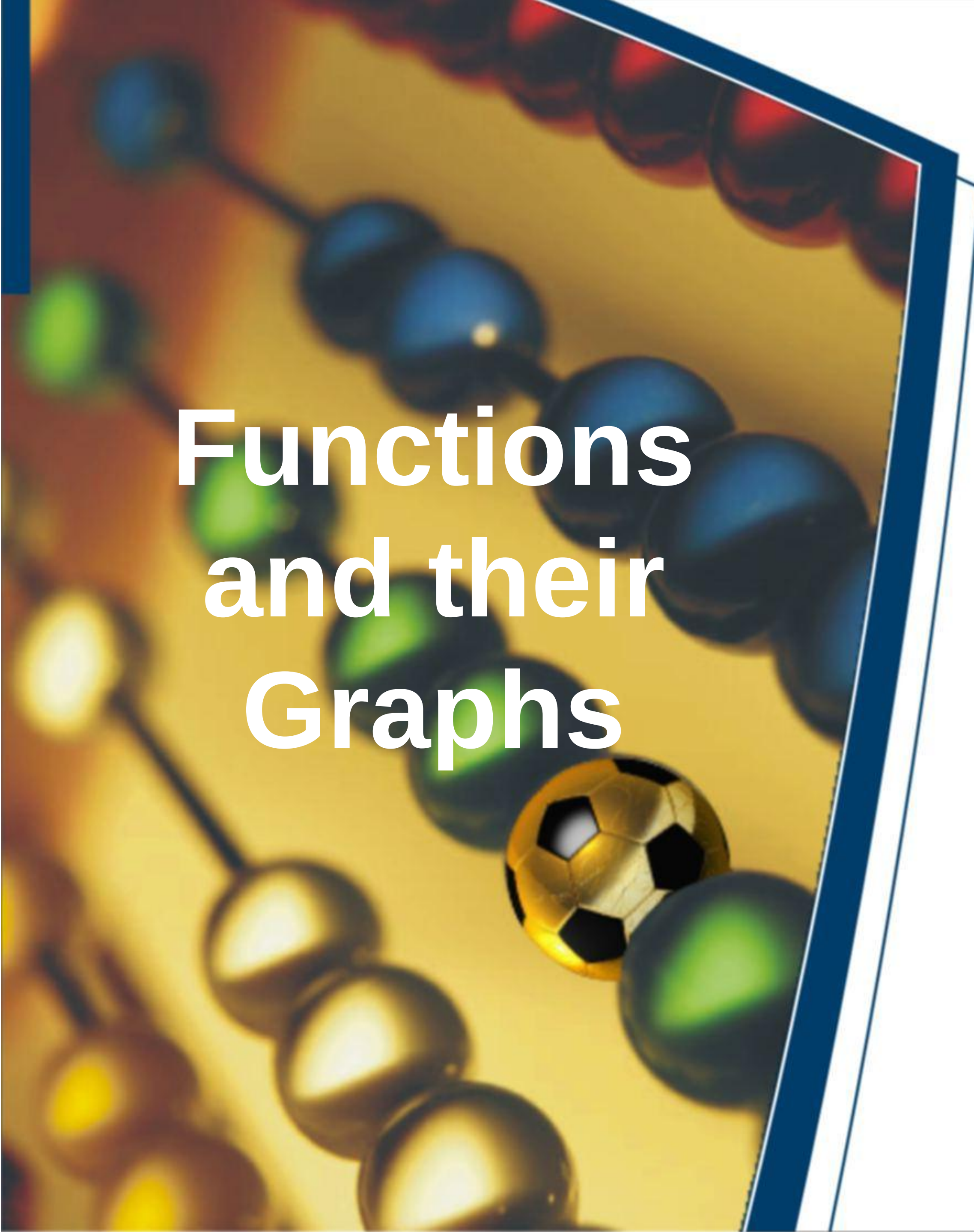

Functions and their Graphs

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Outcomes for this DVD

In this DVD you will :

- Revise the seven basic functions.

  LESSON 1.

- Investigate the effect of parameters.

  LESSON 2.

- Generate new graphs.

  LESSON 3.

Lesson 1

The Seven Basic Functions

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Seven Basic Functions

We revise the following basic functions:

1) The straight line function defined by $y = x$.

2) The parabola function defined by $y = x^2$.

3) The hyperbolic function defined by $y = \frac{1}{x}$.

4) The exponential function defined by $y = a^x$.

5) The sine function defined by $y = \sin x$.

6) The cosine function defined by $y = \cos x$.

7) The tangent function defined by $y = \tan x$.

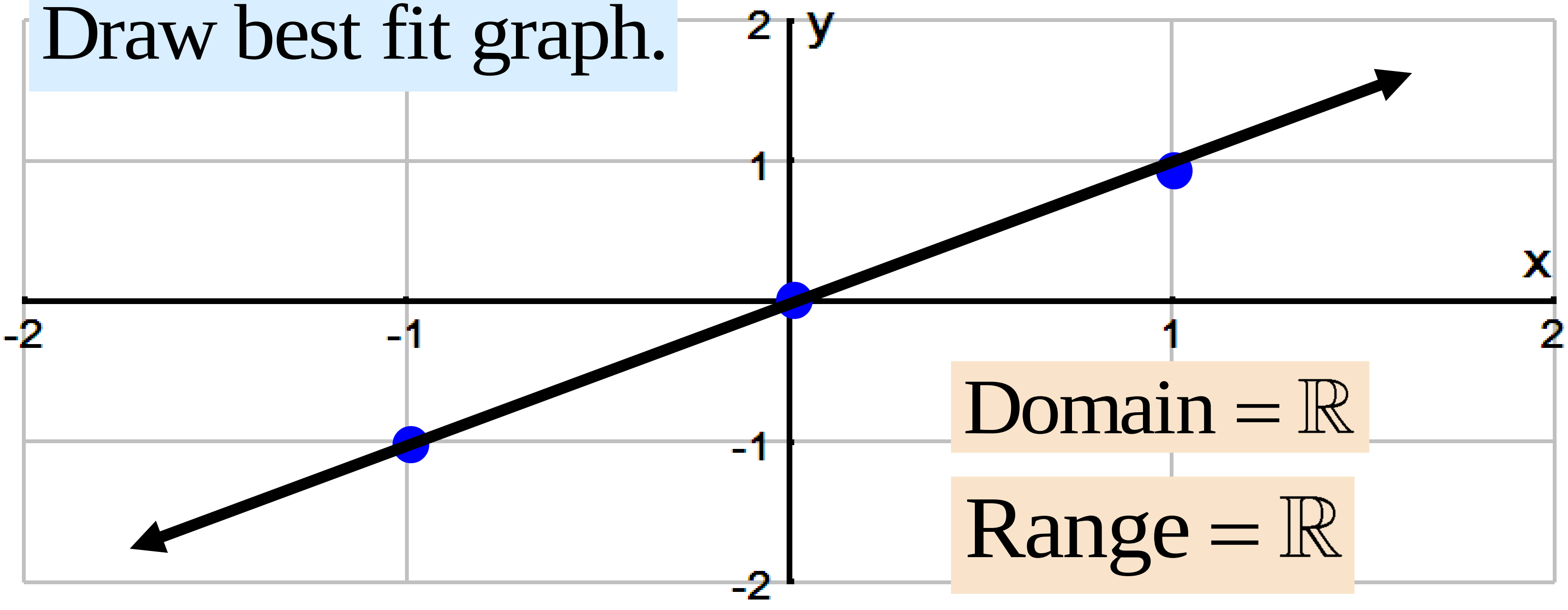
Function defined by $y = x$

Utilize at least the following three ordered pairs:

x	-1	0	1
y	-1	0	1

Plot these points on Cartesian Plane.

Draw best fit graph.



Domain $= \mathbb{R}$

Range $= \mathbb{R}$

Function defined by $y = x^2$

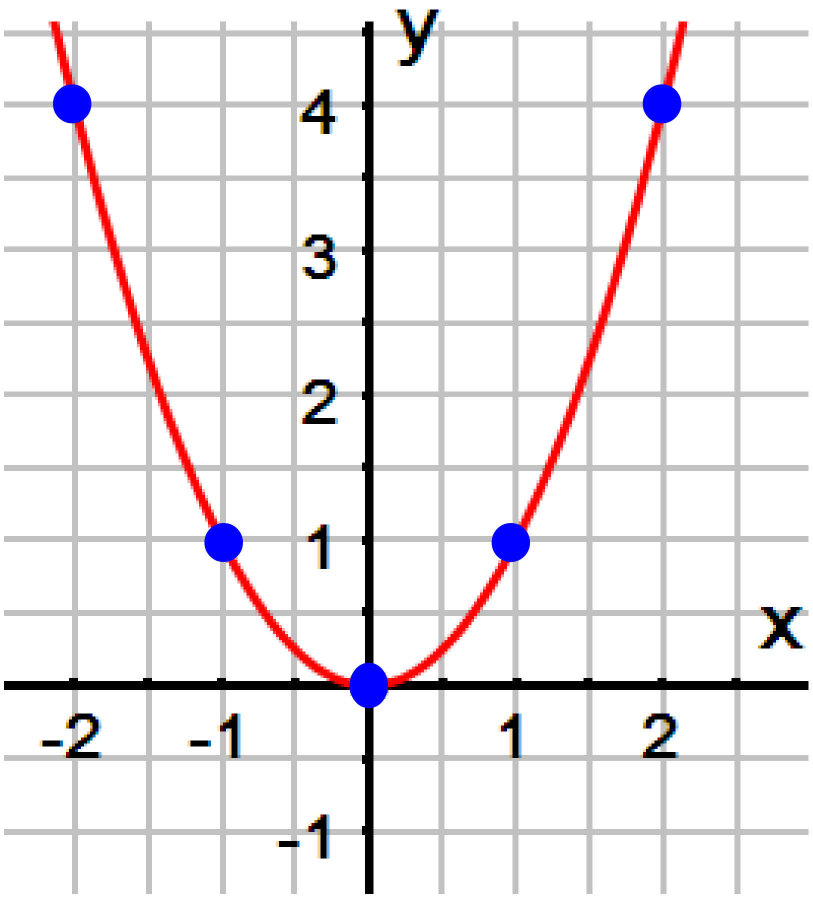
Utilize at least the following five ordered pairs:

x	-2	-1	0	1	2
y	4	1	0	1	4

Plot these points on Cartesian Plane and draw best fit graph.

Domain $= \mathbb{R}$

and Range $= [0; \infty)$



Function defined by $y = \dfrac{1}{x}$

Utilize at least the following six ordered pairs:

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$

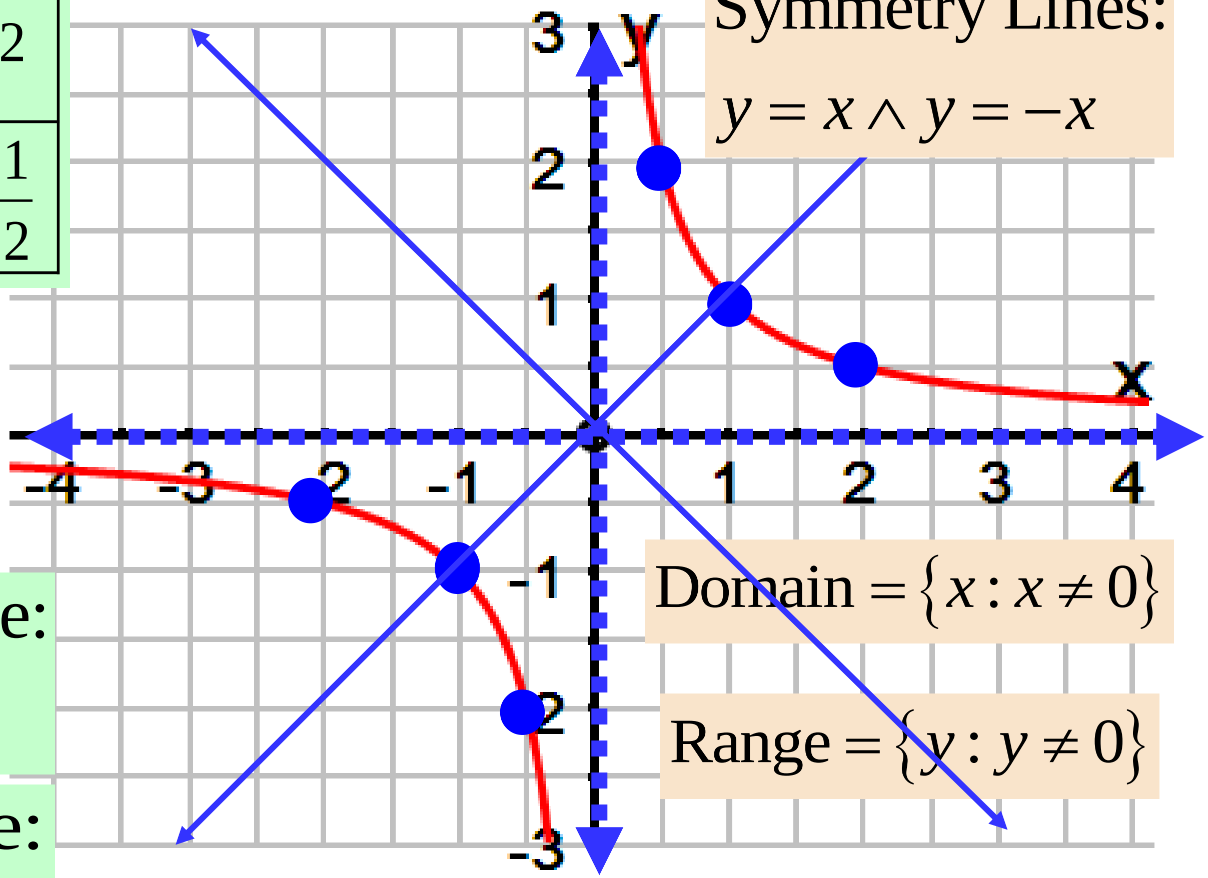
Plot these points on Cartesian Plane and draw graph.

Horizontal Asymptote:

Defined by $y = 0$

Vertical Asymptote:

Defined by $x = 0$



Symmetry Lines:

$y = x \wedge y = -x$

$\text{Domain} = \{x : x \neq 0\}$

$\text{Range} = \{y : y \neq 0\}$

Function defined by $y = a^x; a > 0; a \neq 1$

Select $a = 2$ and utilize at least the following five ordered pairs:

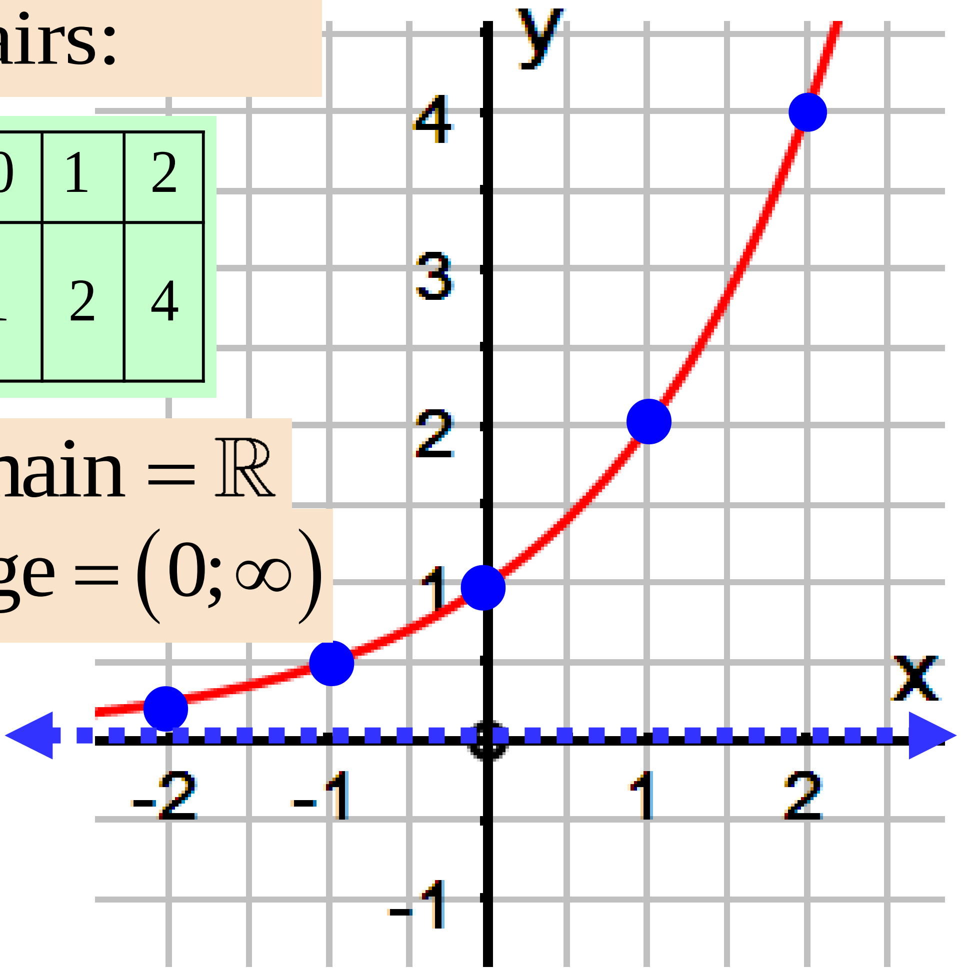
$$y = 2^x$$

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

Plot these points on Cartesian Plane and draw best fit graph.

Domain $= \mathbb{R}$

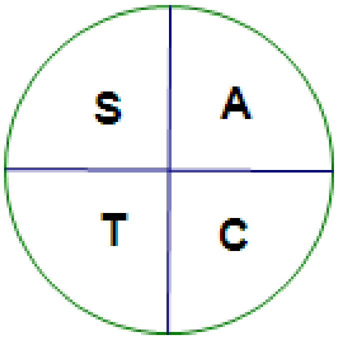
Range $= (0; \infty)$

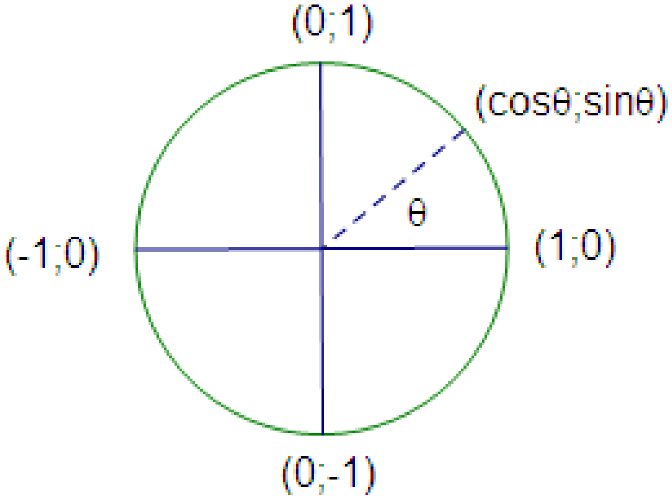


Horizontal Asymptote: Defined by $y = 0$

Finding points on $y = \sin x$

Utilize CAST – diagram and Unit Circle





Complete table:

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0

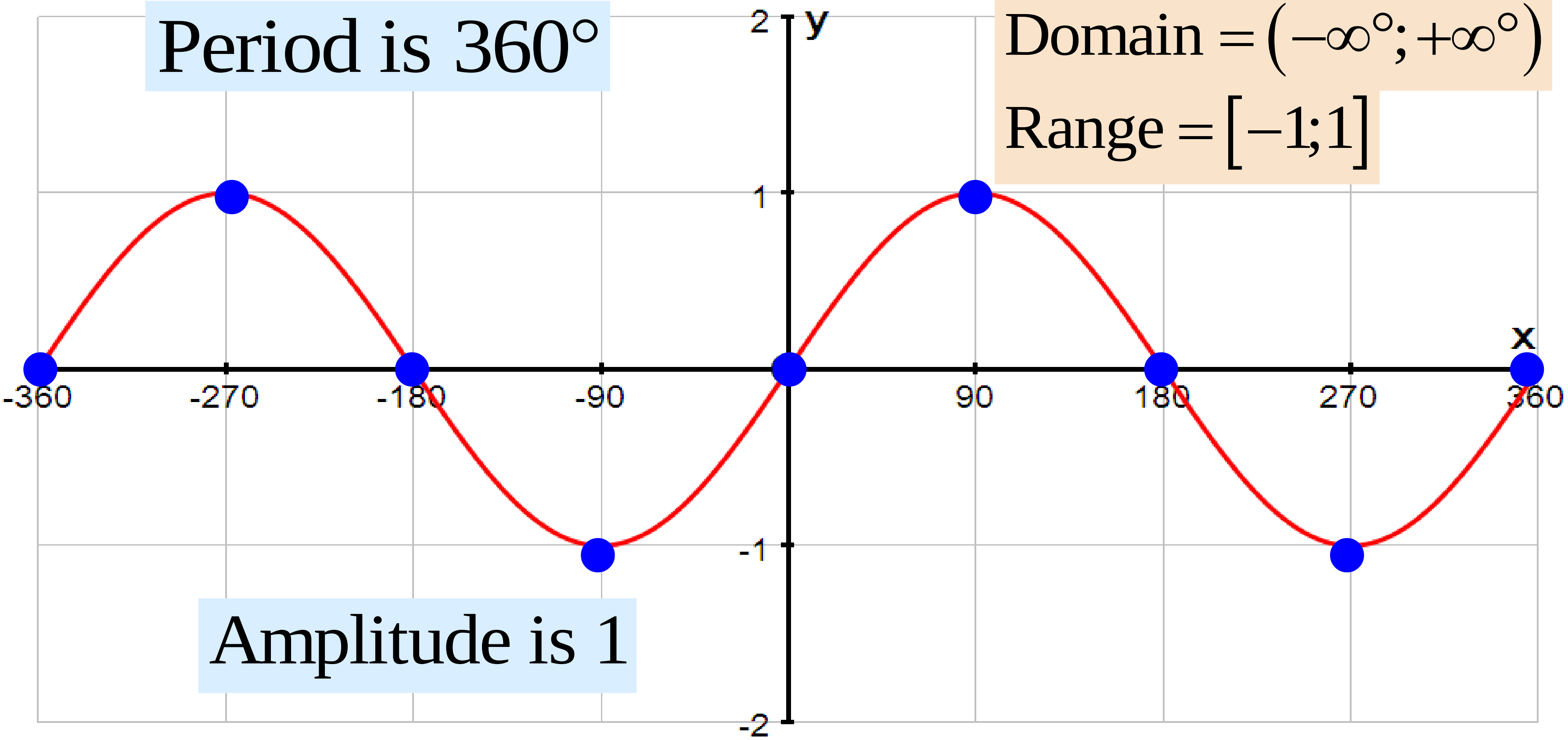
Plot points and sketch $y = \sin x$

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0

Period is 360°

$\text{Domain} = (-\infty°; +\infty°)$

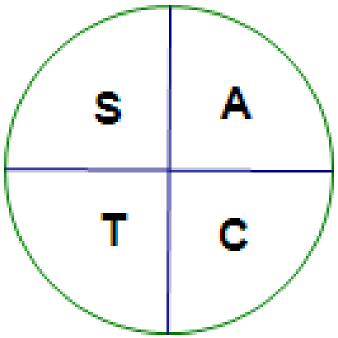
$\text{Range} = [-1; 1]$

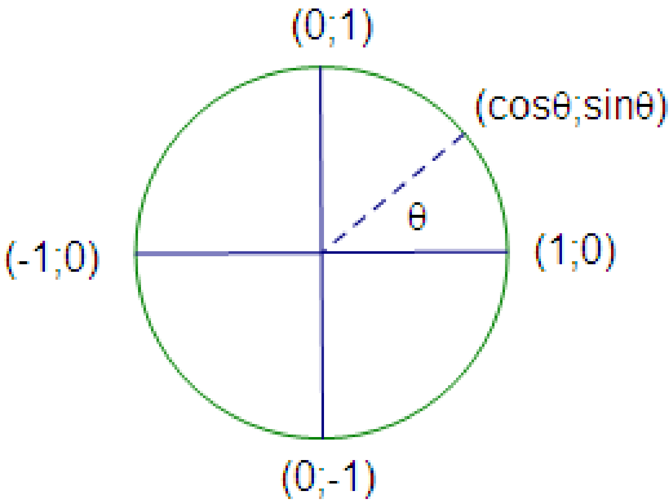


Amplitude is 1

Finding points on $y = \cos x$

Utilize CAST – diagram and Unit Circle



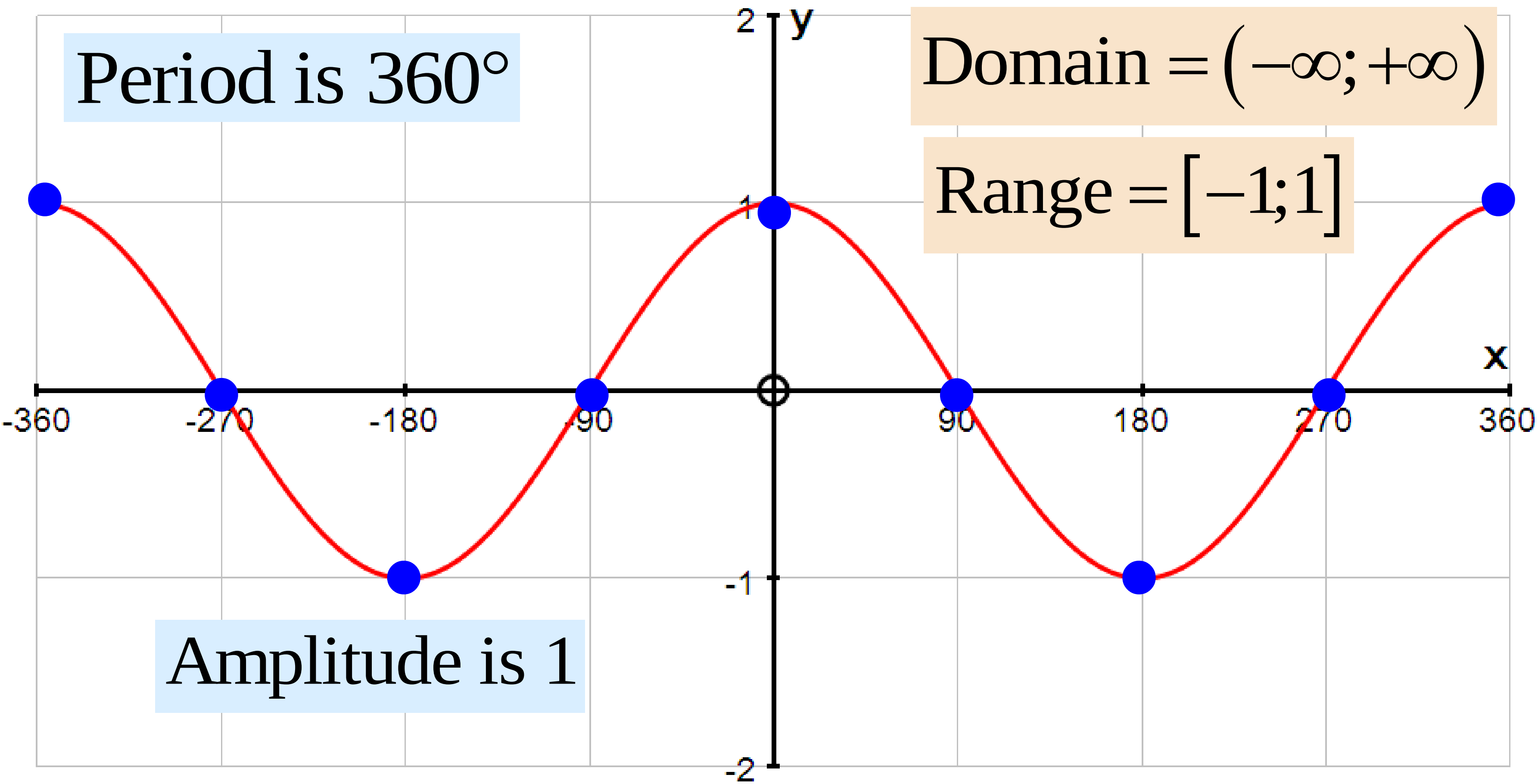


Complete table:

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1

Plot points and sketch $y = \cos x$

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1



Period is 360°

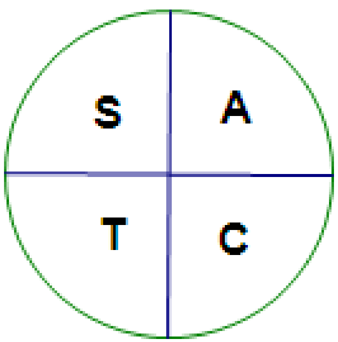
Domain $= (-\infty; +\infty)$

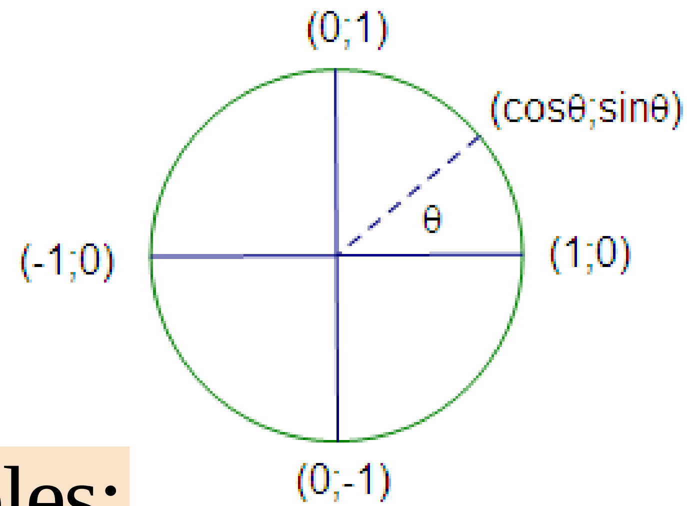
Range $= [-1; 1]$

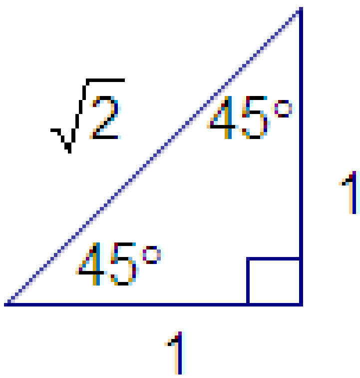
Amplitude is 1

Finding points on $y = \tan x$

Utilize information in the following three diagrams:







Complete tables:

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	$\pm\infty$	0	$\pm\infty$	0	$\pm\infty$	0	$\pm\infty$	0

x	$-315°$	$-225°$	$-135°$	$-45°$	$45°$	$135°$	$225°$	$315°$
y	1	-1	1	-1	1	-1	1	-1

Plot points and sketch $y = \tan x$

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	$\pm\infty$	0	$\pm\infty$	0	$\pm\infty$	0	$\pm\infty$	0

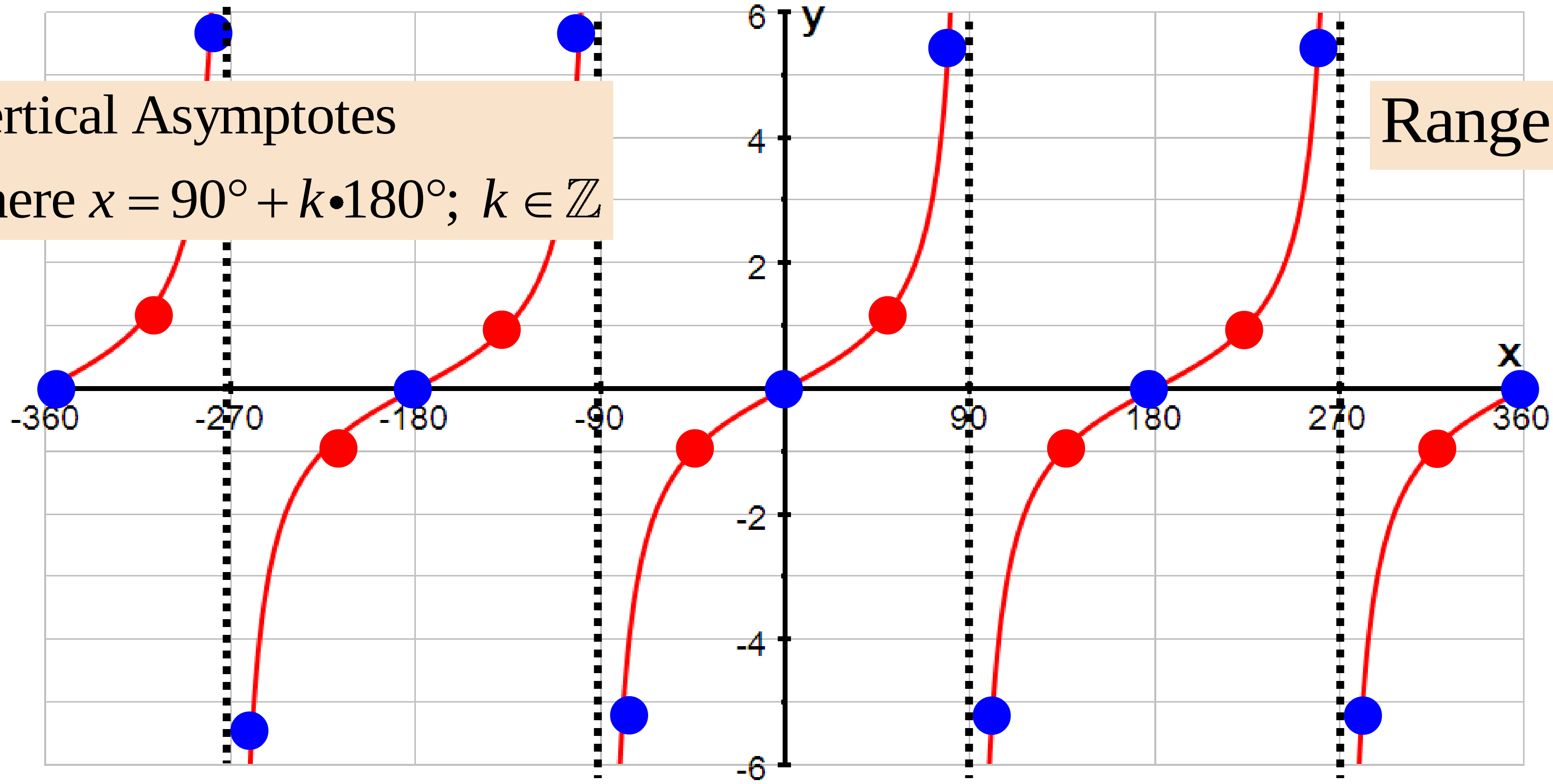
x	$-315°$	$-225°$	$-135°$	$-45°$	$45°$	$135°$	$225°$	$315°$
y	1	-1	1	-1	1	-1	1	-1

Period is 180°

Domain $= \{x : x \neq 90° + k \cdot 180°;\ k \in \mathbb{Z}\}$

Vertical Asymptotes where $x = 90° + k \cdot 180°;\ k \in \mathbb{Z}$

Range $= \mathbb{R}$



Tutorial 1: Basic Functions

Sketch the functions defined by:

1) $y = 3^{-x}$ if $x \in [-2; 2]$.

2) $y = \sin x$ if $x \in [0°; 360°]$

3) $y = \cos x$ if $x \in [-180°; 180°]$

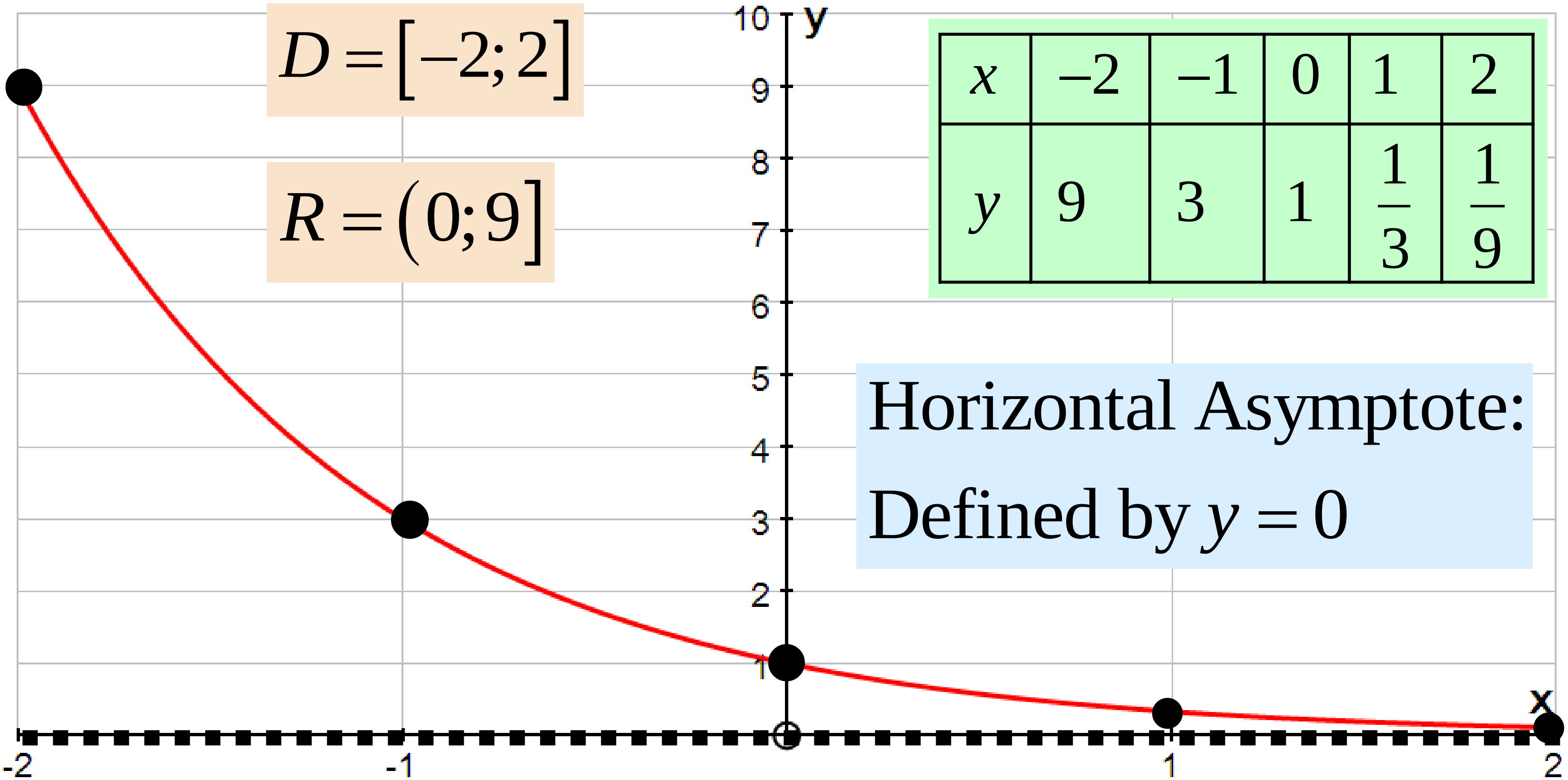
4) $y = \tan x$ if $x \in [-90°; 90°]$

PAUSE DVD

- Do Tutorial 1
- Then View Solutions

Tutorial 1 Problem 1: Suggested Solution

1) Sketch $y = 3^{-x}$ if $x \in [-2;2]$



$D = [-2;2]$

$R = (0;9]$

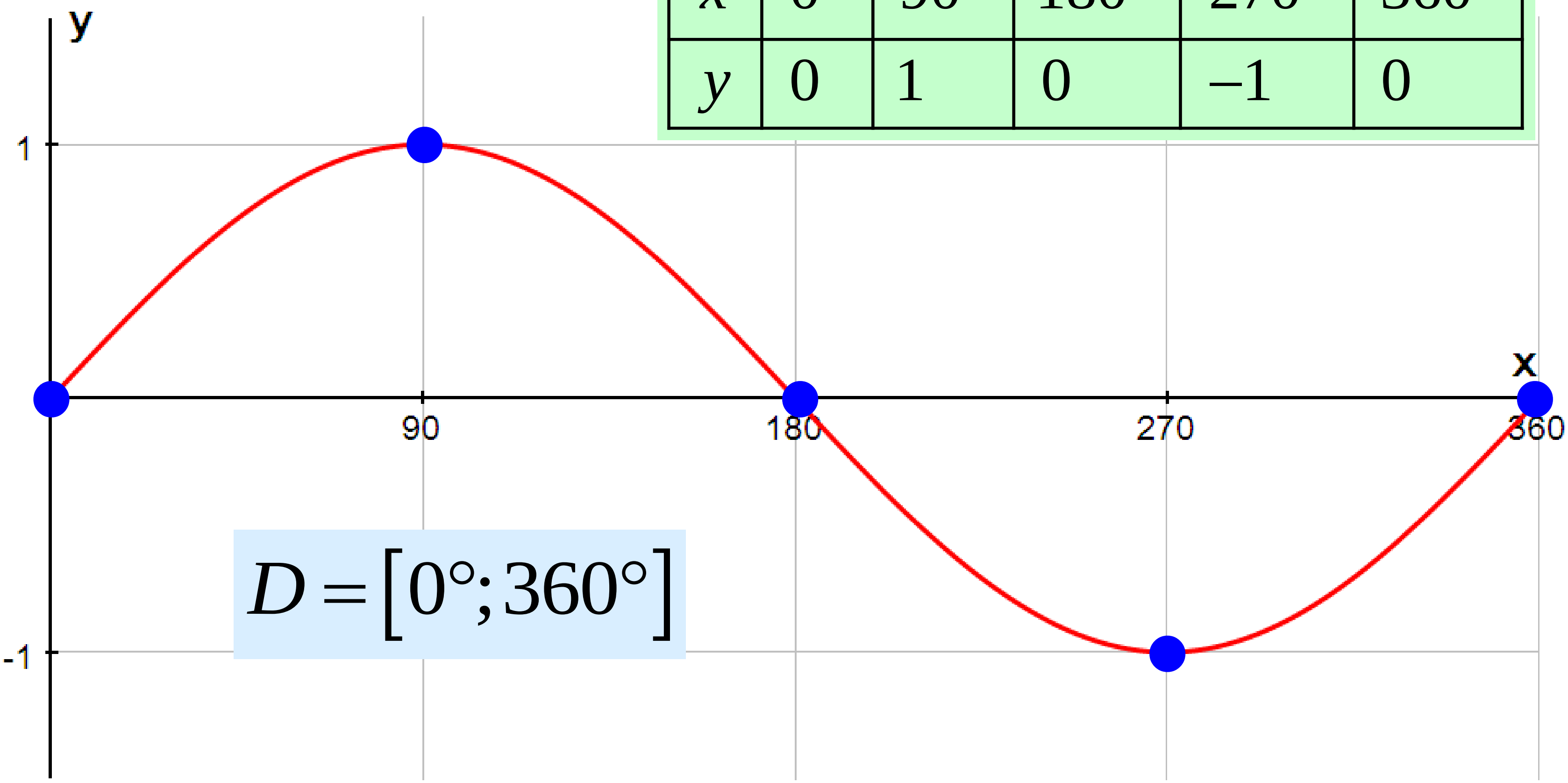
x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

Horizontal Asymptote:
Defined by $y = 0$

Tutorial 1 Problem 2: Suggested Solution

2) Sketch $y = \sin x$ if $x \in [0°;360°]$

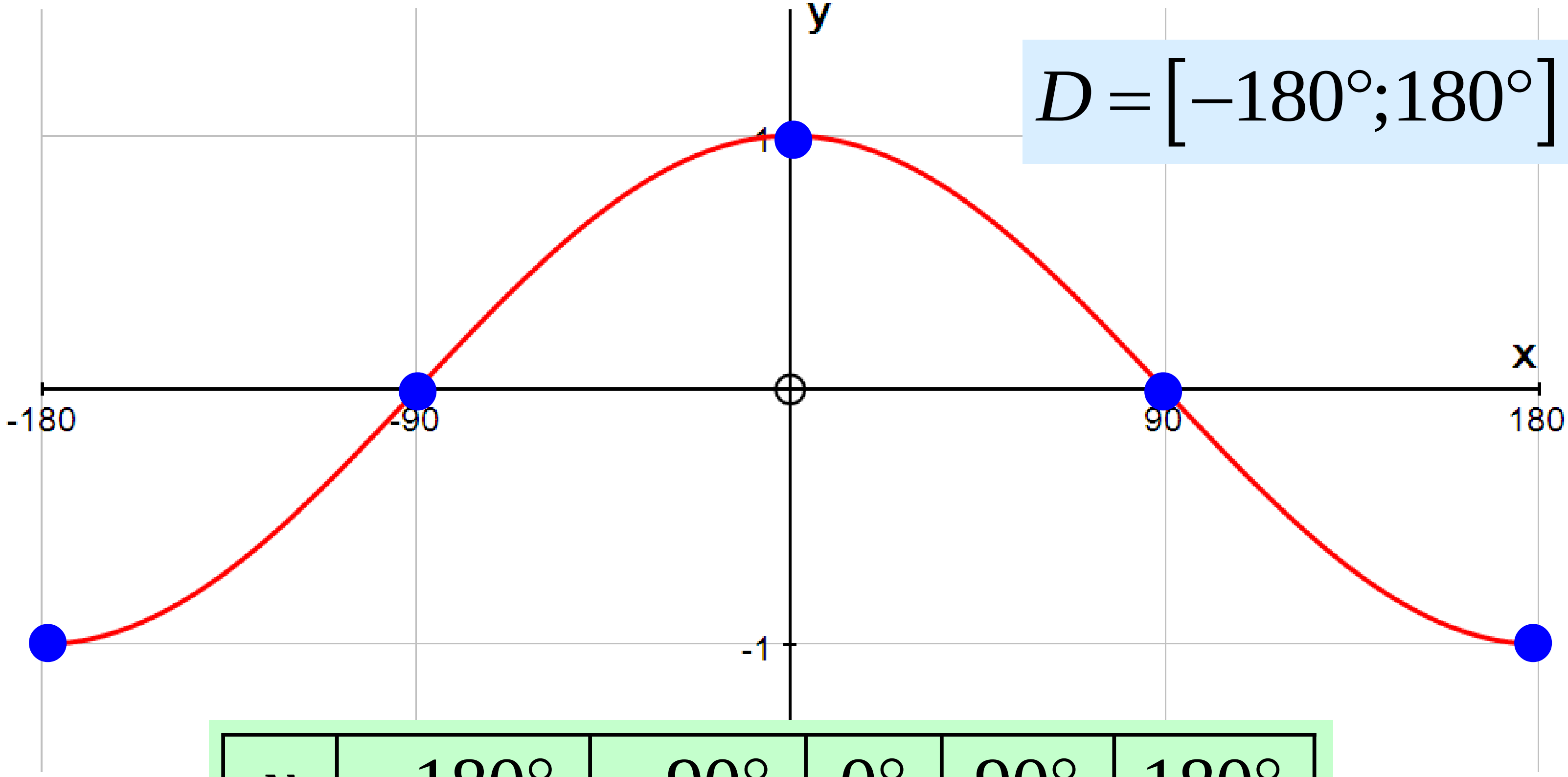
x	0°	90°	180°	270°	360°
y	0	1	0	–1	0



$D = [0°;360°]$

Tutorial 1 Problem 3: Suggested Solution

3) Sketch $y = \cos x$ if $x \in [-180°;180°]$



$D = [-180°;180°]$

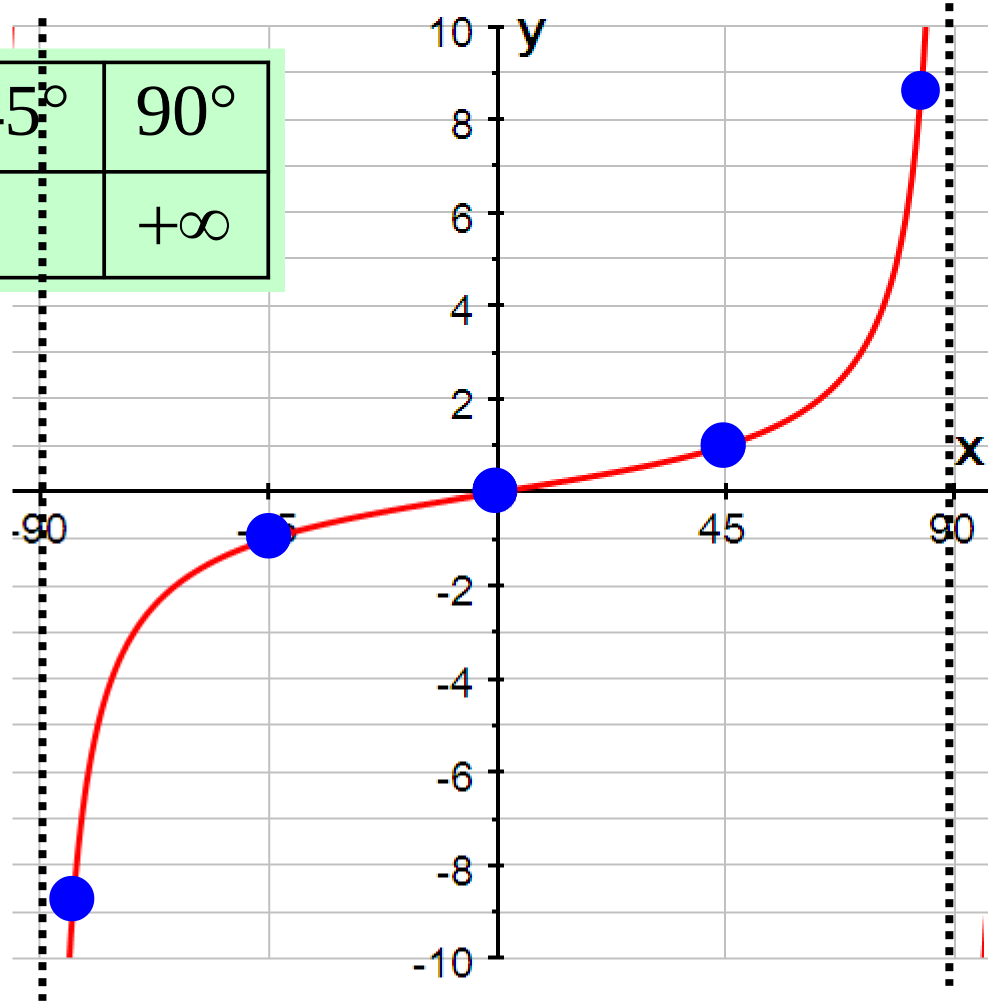
x	$-180°$	$-90°$	$0°$	$90°$	$180°$
y	-1	0	1	0	-1

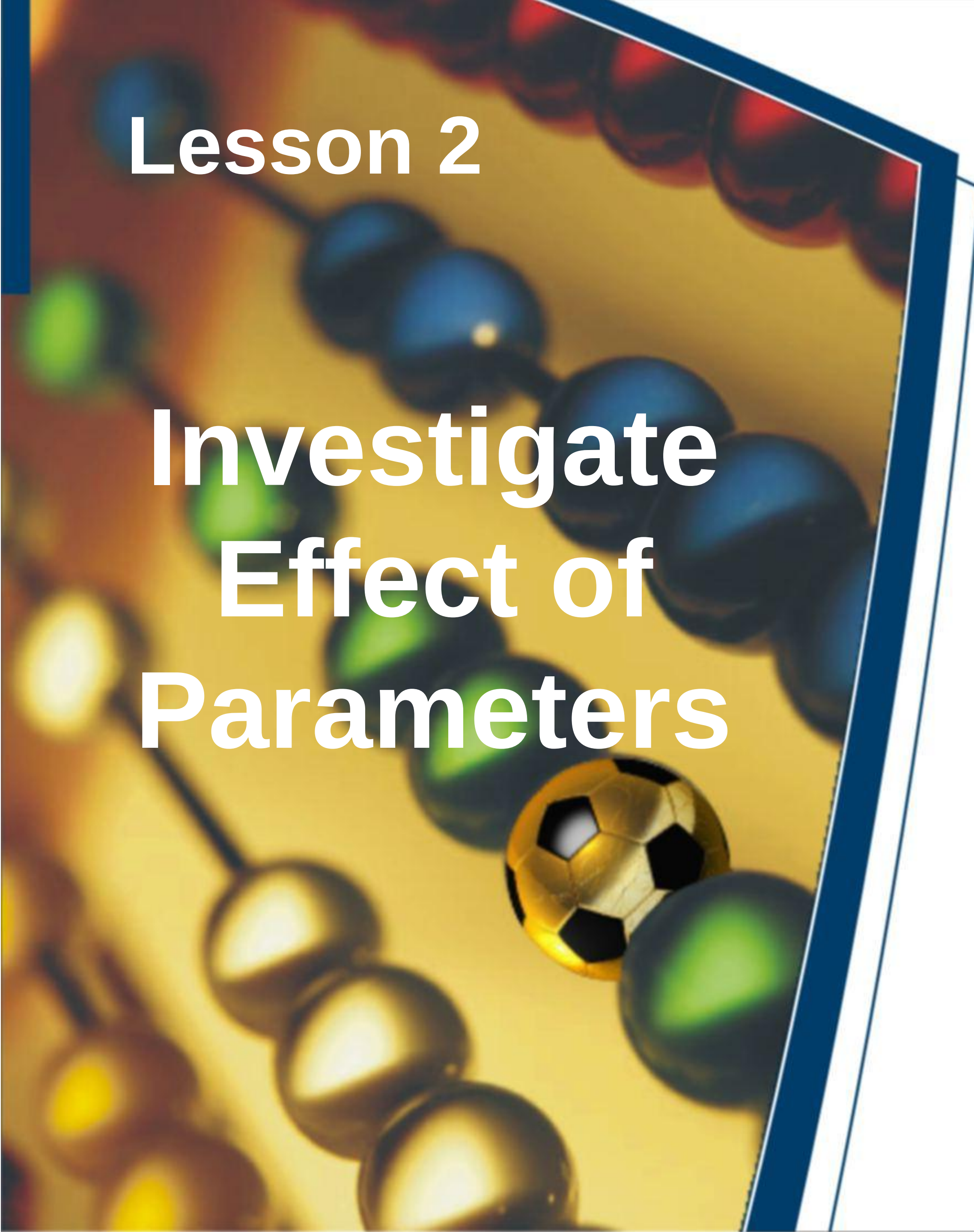
Tutorial 1 Problem 4: Suggested Solution

4) Sketch $y = \tan x$ if $x \in [-90°; 90°]$

x	$-90°$	$-45°$	$0°$	$45°$	$90°$
y	$-\infty$	-1	0	1	$+\infty$

$$D = (-90°; 90°)$$





Lesson 2

Investigate Effect of Parameters

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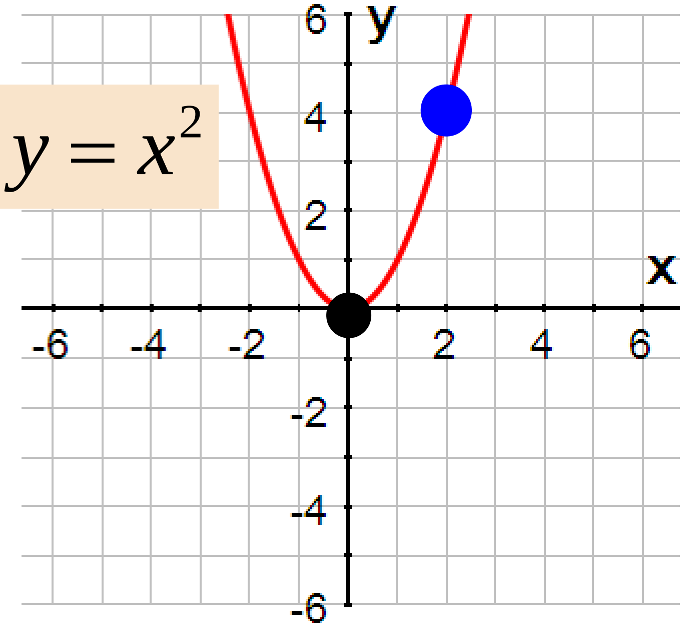
Purpose of Investigation

- Will consider the transformation of points on the seven basic functions defined by $y = f(x)$.
- By applying certain transformations to points on the basic graphs we can obtain points on the graphs of related functions.
- This will give us the ability to sketch the graphs of many related functions quickly by hand.
- It will also enable us to write down defining equations from given graphs.
- Investigate the effect of parameters a, b, c and d on points linked to $y = f(x)$ when basic defining equation is transformed to $y = a \times f\left[b \times (x - c)\right] + d$.

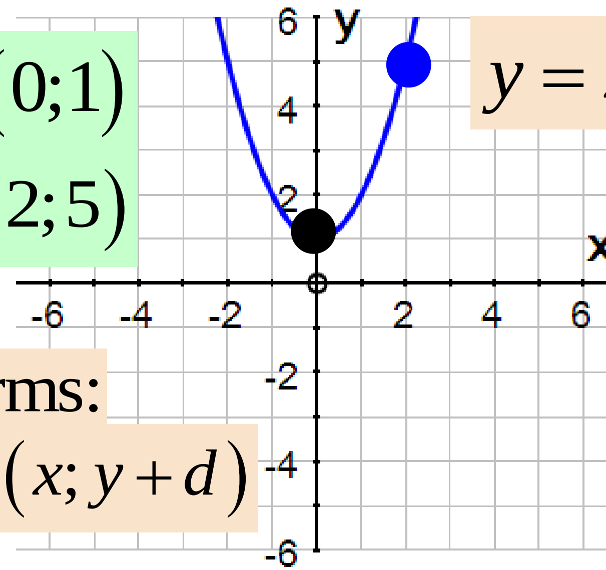
Vertical Shifts (Translations)

$y = x^2$



$(0,0) \rightarrow (0;1)$

$(2;4) \rightarrow (2;5)$



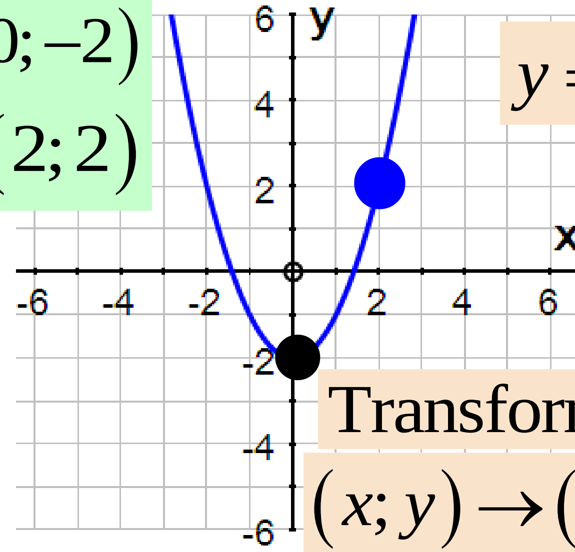
$y = x^2 + 1$

Transforms:

$(x;y) \rightarrow (x;y+d)$

$(0;0) \rightarrow (0;-2)$

$(2;4) \rightarrow (2;2)$



$y = x^2 - 2$

Transforms:

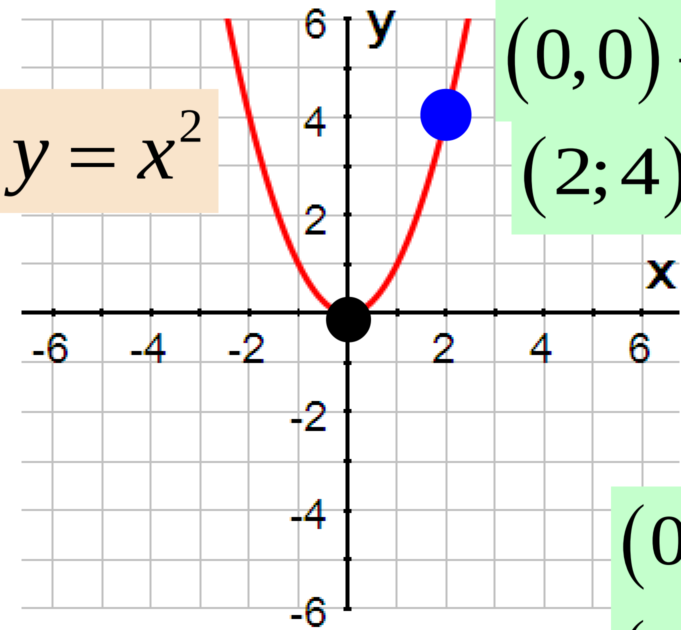
$(x;y) \rightarrow (x;y-d)$

Assume that $d > 0$:

- $y = f(x) + d$, shift the graph of $y = f(x)$ a distance of d units vertically upward.
- $y = f(x) - d$, shift the graph of $y = f(x)$ a distance of d units vertically downward.

Horizontal Shifts (Translations)

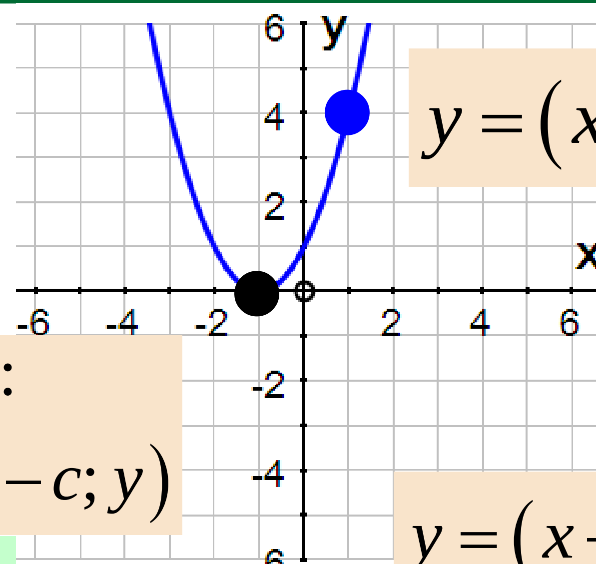
$y = x^2$



$(0,0) \rightarrow (-1;0)$

$(2;4) \rightarrow (1;4)$

$y = (x+1)^2$



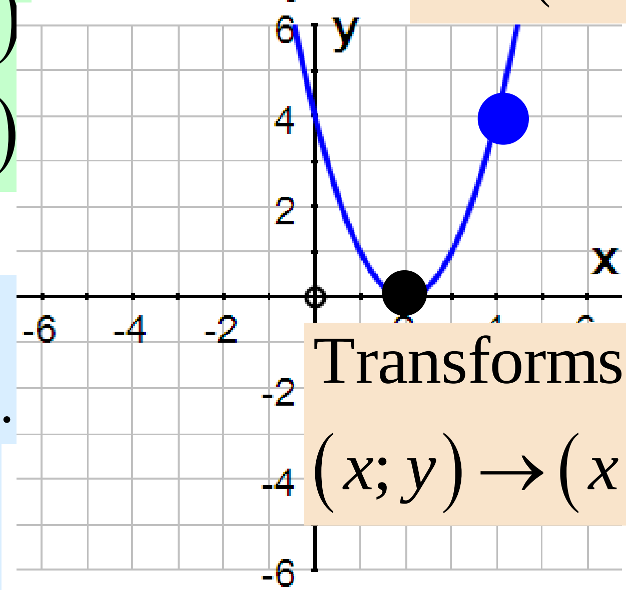
Transforms:

$(x;y) \rightarrow (x-c;y)$

$(0;0) \rightarrow (2;0)$

$(2;4) \rightarrow (4;4)$

$y = (x-2)^2$



Transforms:

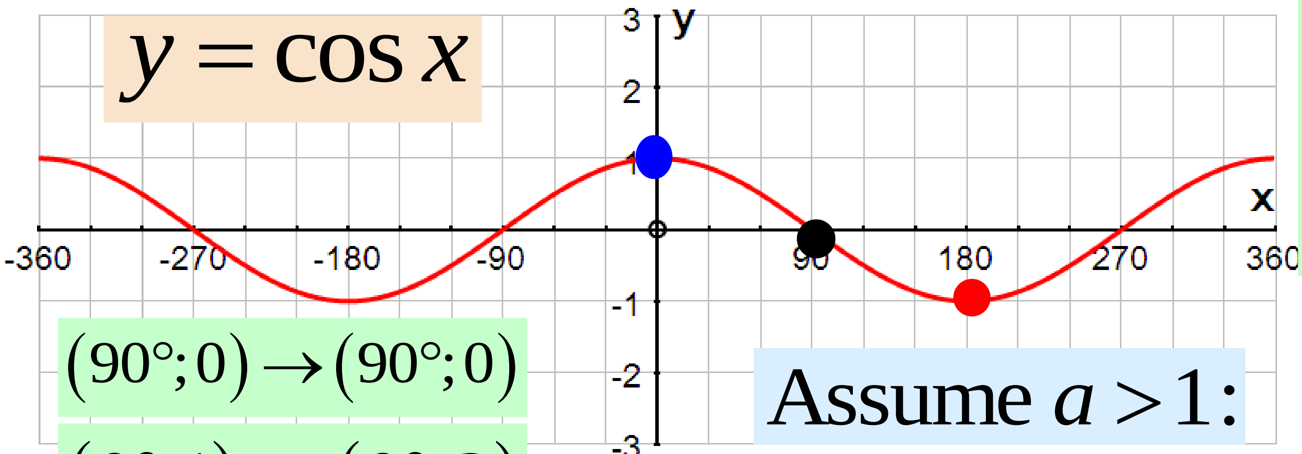
$(x;y) \rightarrow (x+c;y)$

Assume that $c > 0$:

- $y = f(x+c)$, shift the graph of $y = f(x)$ a distance of c units horizontally to the left.
- $y = f(x-c)$, shift the graph of $y = f(x)$ a distance of c units horizontally to the right.

Vertical Stretching

$y = \cos x$



Transforms:

$(x;y)$ on $y = f(x)$ to

$(x;ay)$ on $y = a \bullet f(x)$

$(90°;0) \rightarrow (90°;0)$

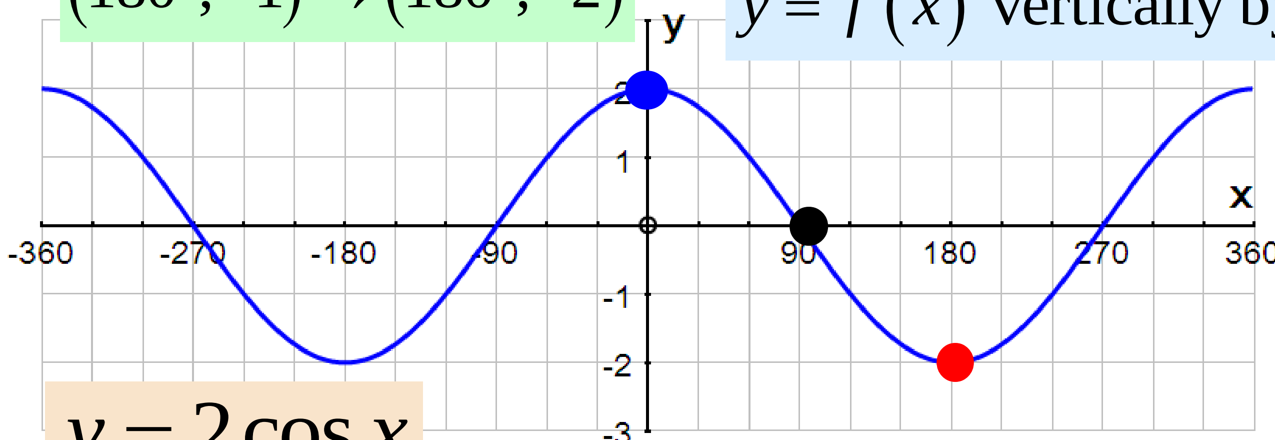
$(0°;1) \rightarrow (0°;2)$

$(180°;-1) \rightarrow (180°;-2)$

Assume $a > 1$:

$y = a \bullet f(x)$ stretch the graph of

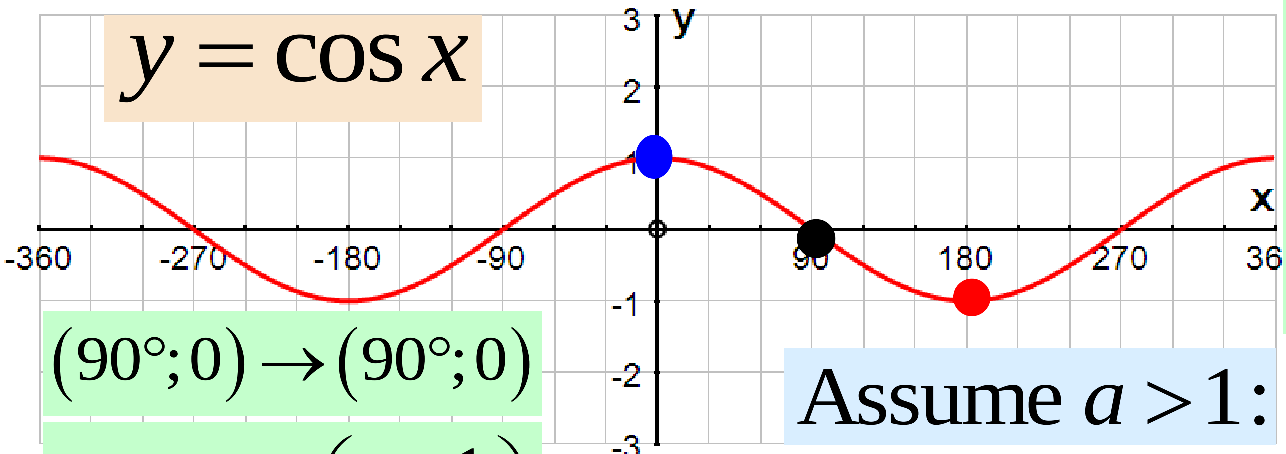
$y = f(x)$ vertically by a factor of a.



$y = 2\cos x$

Vertical Compression

$y = \cos x$



Transforms:

$(x; y)$ on $y = f(x)$ to

$\left(x; \frac{y}{a}\right)$ on $y = \frac{1}{a} \bullet f(x)$

$(90°; 0) \rightarrow (90°; 0)$

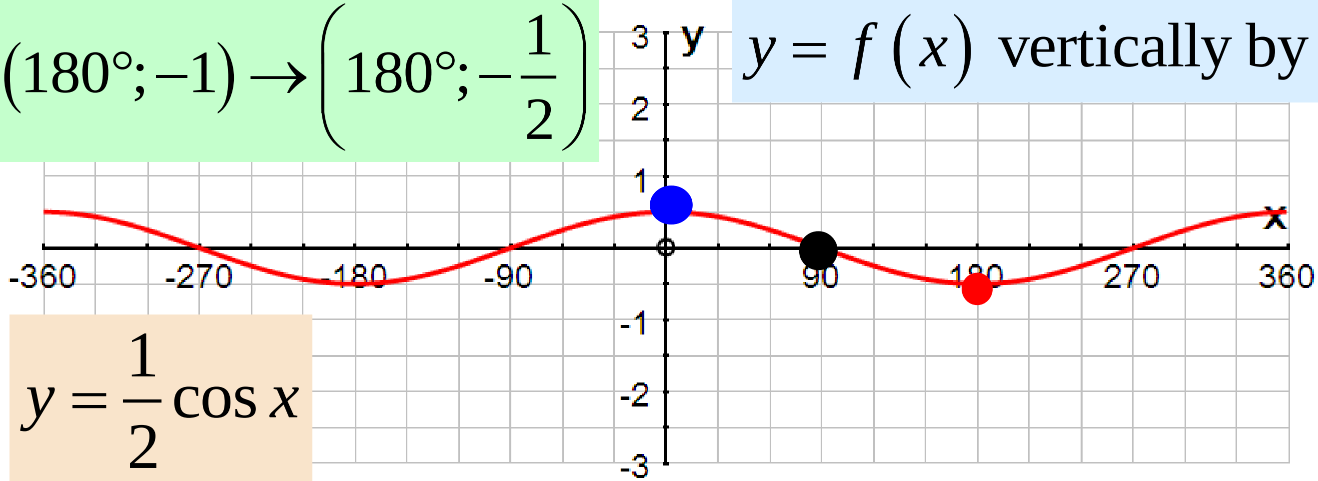
Assume $a > 1$:

$(0°; 1) \rightarrow \left(0°; \frac{1}{2}\right)$

$y = \frac{1}{a} \bullet f(x)$ compress the graph of

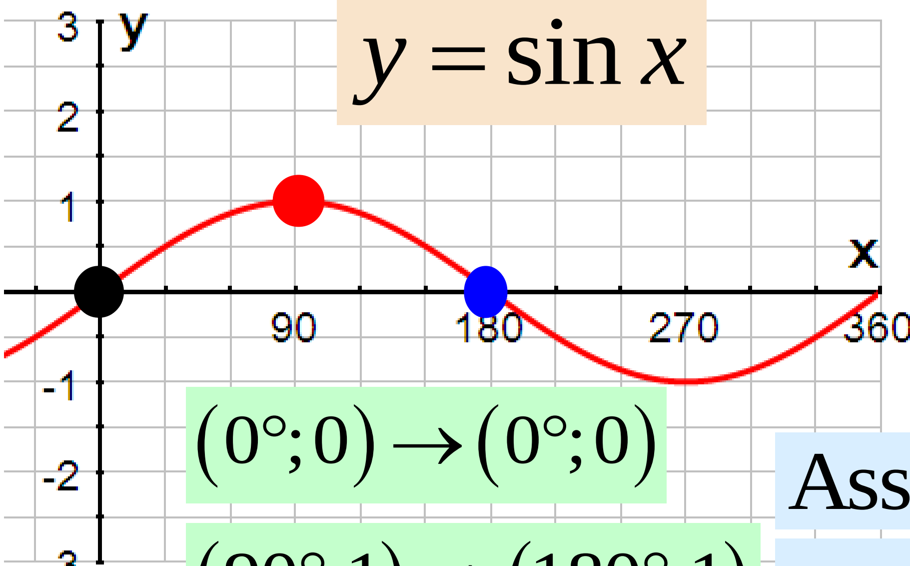
$y = f(x)$ vertically by a factor of a.

$(180°; -1) \rightarrow \left(180°; -\frac{1}{2}\right)$



$y = \frac{1}{2} \cos x$

Horizontal Stretching

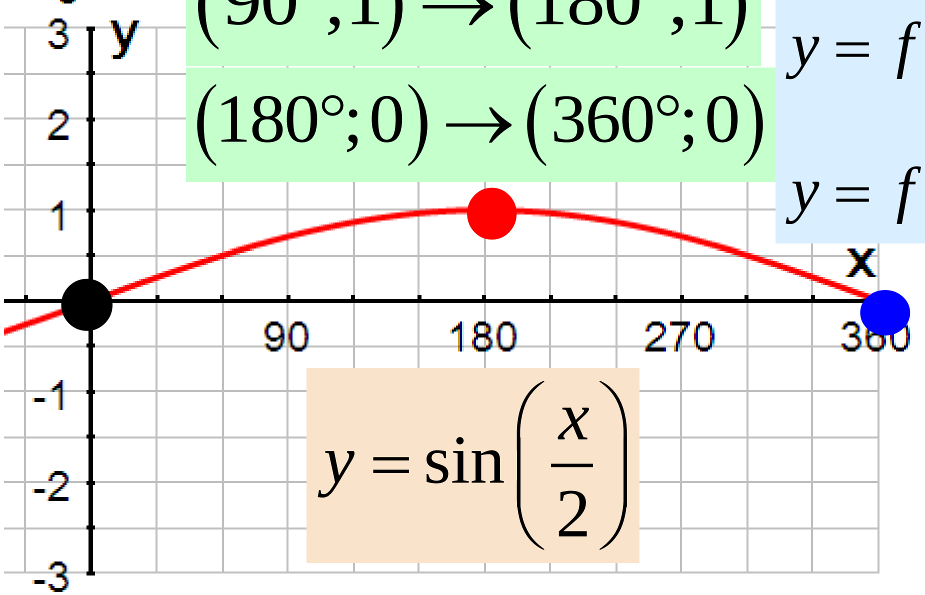


$$y = \sin x$$

$$(0°;0) \rightarrow (0°;0)$$

$$(90°;1) \rightarrow (180°;1)$$

$$(180°;0) \rightarrow (360°;0)$$



$$y = \sin\left(\frac{x}{2}\right)$$

Transforms:

$(x;y)$ on $y = f(x)$ to

$(bx;y)$ on $y = f\left(\frac{1}{b} \bullet x\right)$

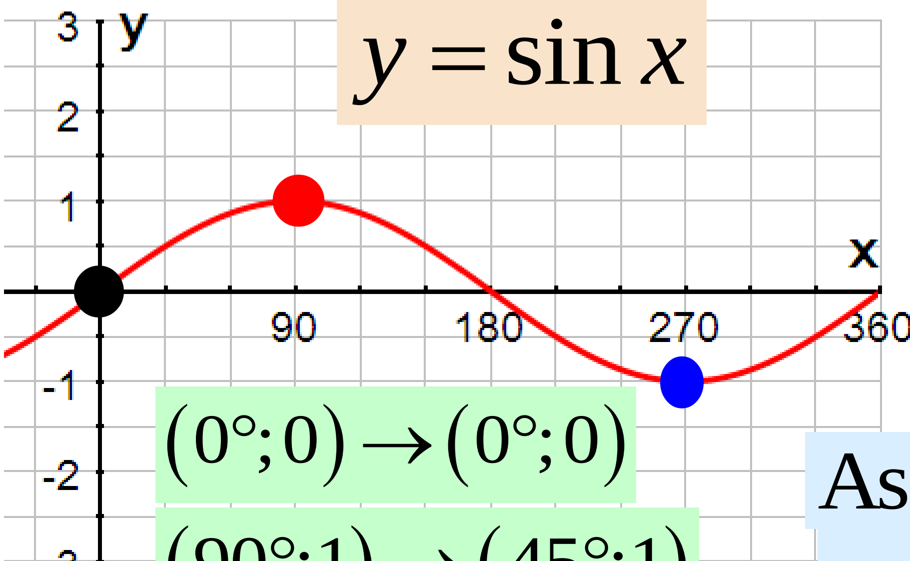
Assume $b > 1$:

$y = f\left(\frac{1}{b} \bullet x\right)$ stretch the graph of

$y = f(x)$ horizontally by a factor of b.

Horizontal Compression

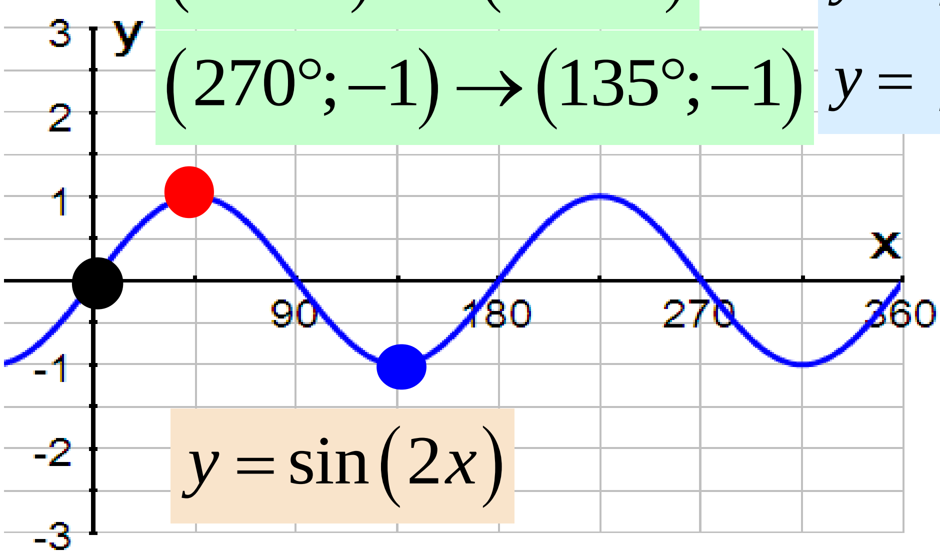
$y = \sin x$



$(0°;0) \to (0°;0)$

$(90°;1) \to (45°;1)$

$(270°;-1) \to (135°;-1)$



$y = \sin(2x)$

Transforms:

$(x;y)$ on $y = f(x)$ to

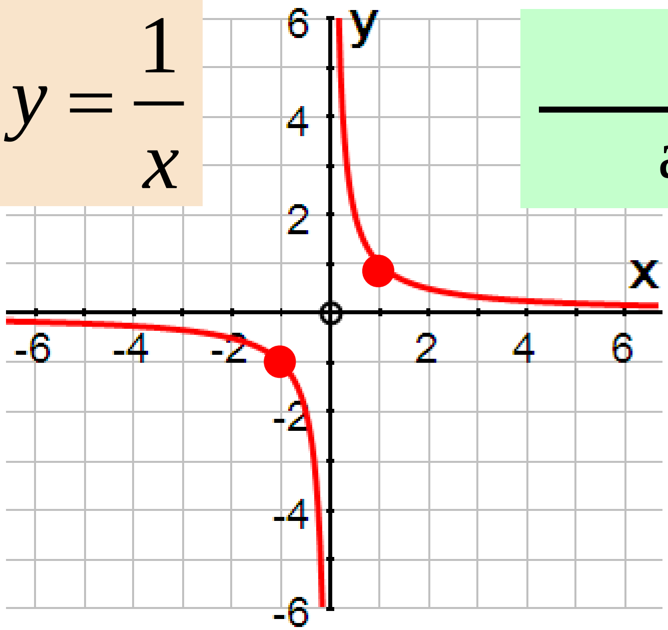
$\left(\frac{x}{b};y\right)$ on $y = f(b \bullet x)$

Assume $b > 1$:

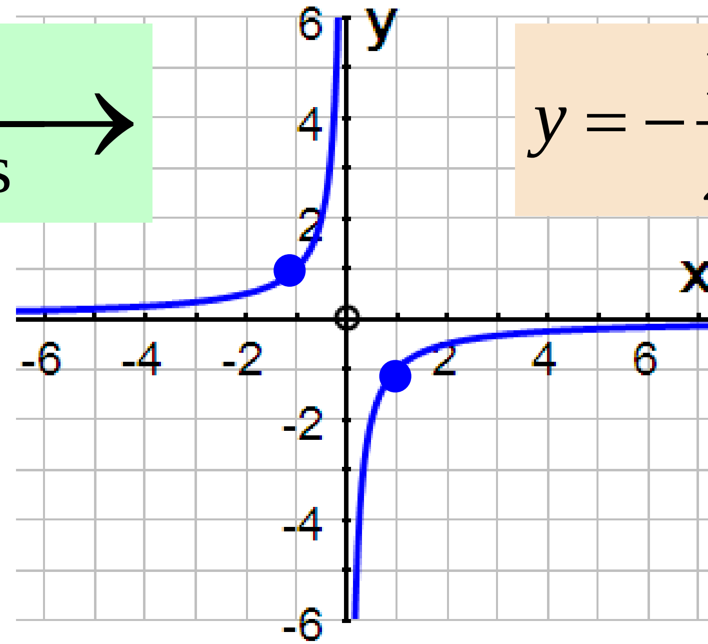
$y = f(b \bullet x)$ compress the graph of $y = f(x)$ horizontally by a factor of b.

Reflection about x-axis

$$y = \frac{1}{x}$$



Reflect $\longrightarrow$ about $x-$axis



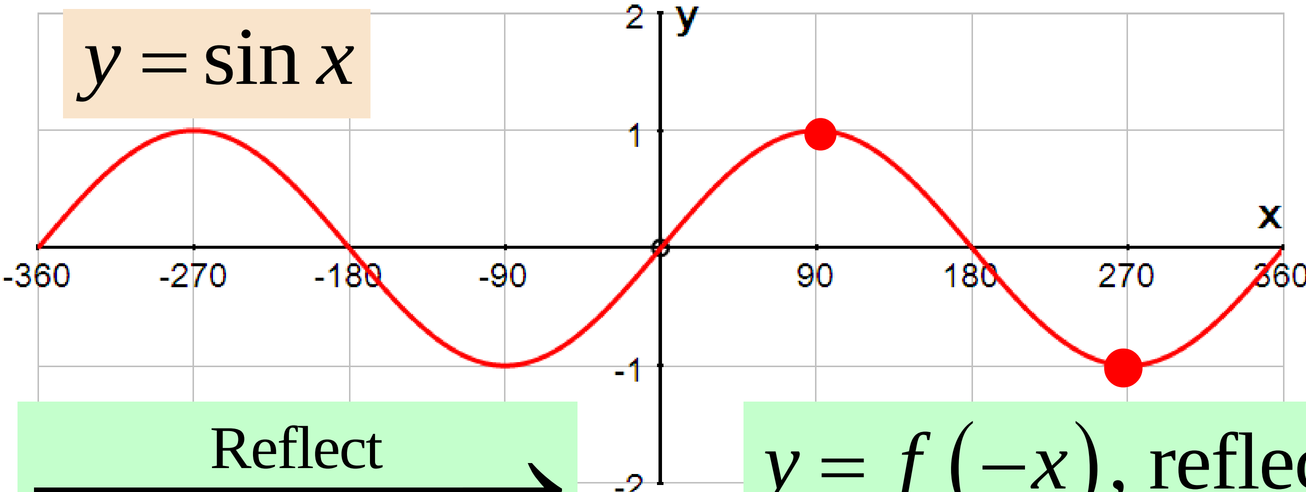
$$y = -\frac{1}{x}$$

$y = -f(x)$, reflect the graph of $y = f(x)$ about the $x-$axis.

Transforms:

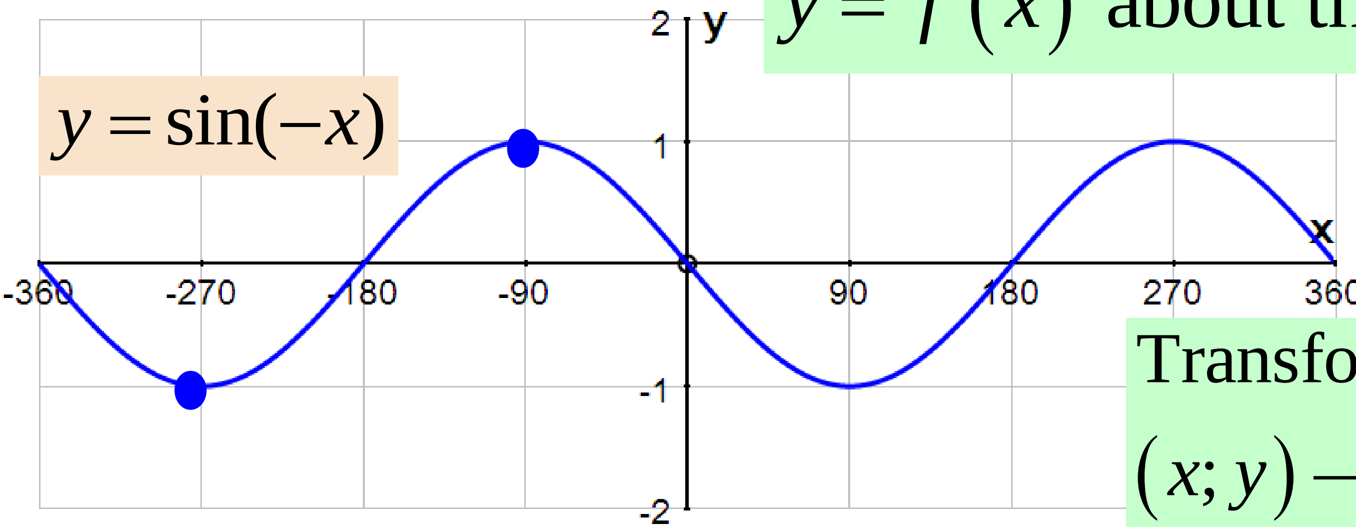
$$(x; y) \rightarrow (x; -y)$$

Reflection about y-axis



$\xrightarrow[\text{about } y-\text{axis}]{\text{Reflect}}$

$y = f(-x)$, reflect the graph of $y = f(x)$ about the $y-$axis.



Transforms:

$(x; y) \to (-x; y)$

Combining a horizontal Shift and a vertical Shift

Consider the function defined by

$$y=\frac{1}{x-2}+3$$

Horizontal Shift by 2 units to the right.

Vertical Shift by 3 units upward.

Select $\left(4;\frac{1}{4}\right)$ on $y=\frac{1}{x}$

Check if $\left(6;3\frac{1}{4}\right)$ satisfies $y=\frac{1}{x-2}+3$

- Is the order in which the two parameters are applied important?

Any order!

$$\left(4;\frac{1}{4}\right)\xrightarrow[\text{by 2 units to the right}]{\text{Horizontal Shift}}\left(6;\frac{1}{4}\right)\xrightarrow[\text{by 3 units upward}]{\text{Vertical Shift}}\left(6;3\frac{1}{4}\right)$$

$$\left(4;\frac{1}{4}\right)\xrightarrow[\text{by 3 units upward}]{\text{Vertical Shift}}\left(4;3\frac{1}{4}\right)\xrightarrow[\text{by 2 units to the right}]{\text{Horizontal Shift}}\left(6;3\frac{1}{4}\right)$$

Combining a Vertical Stretch and a Horizontal Compression

Consider the function defined by

$$y = 3 \times \sin(2x)$$

Vertical Stretch by factor 3.
Horizontal Compression by factor 2.

Select $(90°;1)$ on $y = \sin x$

Is the order in which the two parameters are applied important?

Check if $(45°;3)$ satisfies $y = 3\sin(2x)$

Any order!

$$(90°;1) \xrightarrow[\text{by factor 3}]{\text{Vertical Stretch}} (90°;3) \xrightarrow[\text{by factor 2}]{\text{Horizontal Compression}} (45°;3)$$

$$(90°;1) \xrightarrow[\text{by factor 2}]{\text{Horizontal Compression}} (45°;1) \xrightarrow[\text{by factor 3}]{\text{Vertical Stretch}} (45°;3)$$

Combining a Vertical Stretch and a Vertical Shift

Consider the function defined by

$y = 2 \cdot x^2 + 3$

Vertical Stretch by factor 2.

Vertical Shift by 3 units upward.

Is the order in which the two parameters are applied important?

Select $(2;4)$ on $y = x^2$

Check if $(2;14)$ satisfies $y = 2 \bullet x^2 + 3$

Check if $(2;11)$ satisfies $y = 2 \bullet x^2 + 3$

Stretch before Shift!

$$(2;4) \xrightarrow[\text{by 3 units upward}]{\text{Vertical Shift}} (2;7) \xrightarrow[\text{by factor 2}]{\text{Vertical Stretch}} (2;14)$$

$$(2;4) \xrightarrow[\text{by factor 2}]{\text{Vertical Stretch}} (2;8) \xrightarrow[\text{by 3 units upward}]{\text{Vertical Shift}} (2;11)$$

Tutorial 2: Effect of Parameters

Complete the following table:

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
1.	$y = x$	$(2;2)$	$y = 3x + 2$	
2.	$y = x^2$	$(3;9)$	$y = (x-2)^2 - 3$	
3.	$y = 3^x$	$(2;9)$	$y = 2 \times 3^{x+2}$	
4.	$xy = 1$	$(-1;-1)$	$y = \dfrac{3}{x-2}$	
5.	$y = \sin x$	$(90°;1)$	$y = \sin(2x - 90°)$	
6.	$y = \cos x$	$(180°;-1)$	$y = 3\cos(2x)$	
7.	$y = \tan x$	$(180°;0)$	$y = \tan(3x) - 2$	

PAUSE DVD

- Do Tutorial 2
- Then View Solutions

Tutorial 2 Problem 1: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
1.	$y = x$	$(2;2)$	$y = 3x + 2$	$(2;8)$

- First apply Vertical Stretch by factor 3.
  Followed by Vertical Shift by 2 units upward.

- $(2;2) \xrightarrow[\text{by factor 3}]{\text{Vertical Stretch}} (2;6) \xrightarrow[\text{by 2 units upward}]{\text{Vertical Shift}} (2;8)$

Check if $(2;8)$ satisfies $y = 3 \bullet x + 2$

Tutorial 2 Problem 2: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
2.	$y = x^2$	$(3;9)$	$y = (x-2)^2 - 3$	$(5;6)$

- Vertical Shift by 3 units downward and Horizontal Shift by 2 units to right can be applied in any order ($\therefore$ Simultaneously).

- $$(3;9) \xrightarrow[\text{Horizontal Shift by 2 units to the right}]{\text{Vertical Shift by 3 units downward}} (5;6)$$

Check if $(5;6)$ satisfies $y = (x-2)^2 - 3$

Tutorial 2 Problem 3: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
3.	$y=3^x$	$(2;9)$	$y=2\times 3^{x+2}$	$(0;18)$

- Transformations operates in different directions and can thus be applied in any order ($\therefore$ Simultaneously).

- $(2;9) \xrightarrow[\text{Horizontal Shift by 2 units to left}]{\text{Vertical Stretch by factor 2}} (0;18)$

Check if $(0;18)$ satisfies $y=2\times 3^{x+2}$

Tutorial 2 Problem 4: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
4.	$xy = 1$	$(-1;-1)$	$y = \dfrac{3}{x-2}$	$(1;-3)$

- Transformations operates in different directions and can thus be applied in any order ($\therefore$ Simulataneously).

- $$(-1;-1) \xrightarrow[\text{Horizontal Shift by 2 units to right}]{\text{Vertical Stretch by factor 3}} (1;-3)$$

Check if $(1;-3)$ satisfies $y = \dfrac{3}{x-2}$

Tutorial 2 Problem 5: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
5.	$y = \sin x$	$(90°;1)$	$y = \sin(2x - 90°)$ $y = \sin\left[2(x - 45°)\right]$	$(90°;1)$

- First apply Horizontal Compression by factor 2.
  Followed by Horizontal Shift by 45° to right.

- $(90°;1) \xrightarrow[\text{by factor 2}]{\text{Horizontal Compression}} (45°;1) \xrightarrow[\text{by } 45° \text{ to right}]{\text{Horizontal Shift}} (90°;1)$

Check if $(90°;1)$ satisfies $y = \sin(2x - 90°)$

Tutorial 2 Problem 6: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
6.	$y = \cos x$	$(180°;-1)$	$y = 3\cos(2x)$	$(90°;-3)$

- Order of transformations not important.
  $\therefore$ Apply simultaneously.

- $(180°;-1) \xrightarrow[\text{Horizontal Compression by factor 2}]{\text{Vertical Stretch by factor 3}} (90°;-3)$

Check if $(90°;-3)$ satisfies $y = 3\cos(2x)$

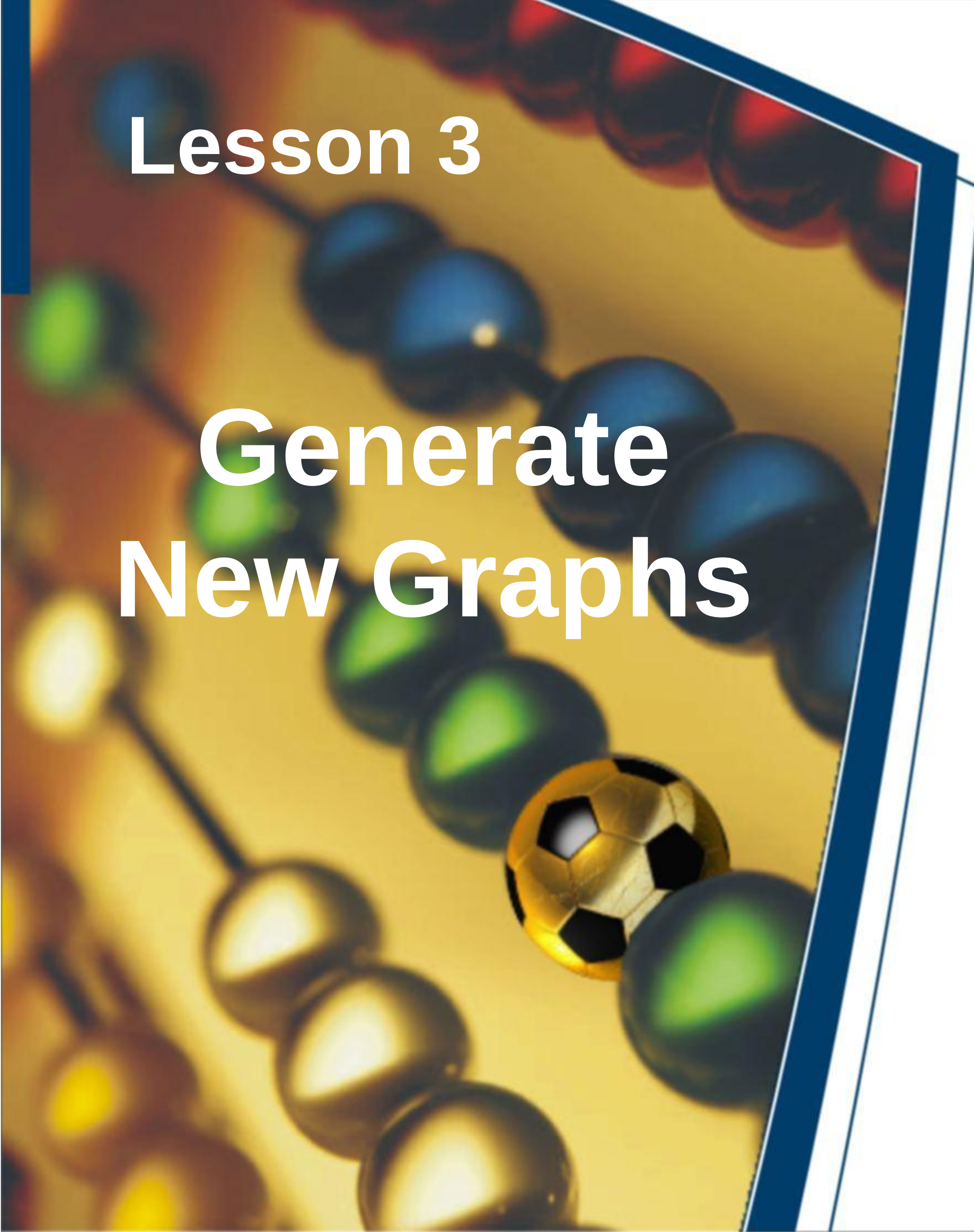
Tutorial 2 Problem 7: Suggested Solution

	Defining equation of basic function	Point on basic function	Defining equation of transformed function	Transformation of point on basic function
7.	$y = \tan x$	$(180°; 0)$	$y = \tan(3x) - 2$	$(60°; -2)$

- Transformations operates in different directions and can thus be applied in any order ($\therefore$ Simultaneously).

$$\bullet (180°; 0) \xrightarrow[\text{Horizontal Compression by factor 3}]{\text{Vertical Shift by 2 units downward}} (60°; -2)$$

Check if $(60°; -2)$ satisfies $y = \tan(3x) - 2$



Lesson 3

Generate New Graphs

Nelson Mandela Metropolitan University

for tomorrow

NCS Mathematics DVD Series

GOVAN MBEKI MATHEMATICS DEVELOPMENT UNIT

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General Comments

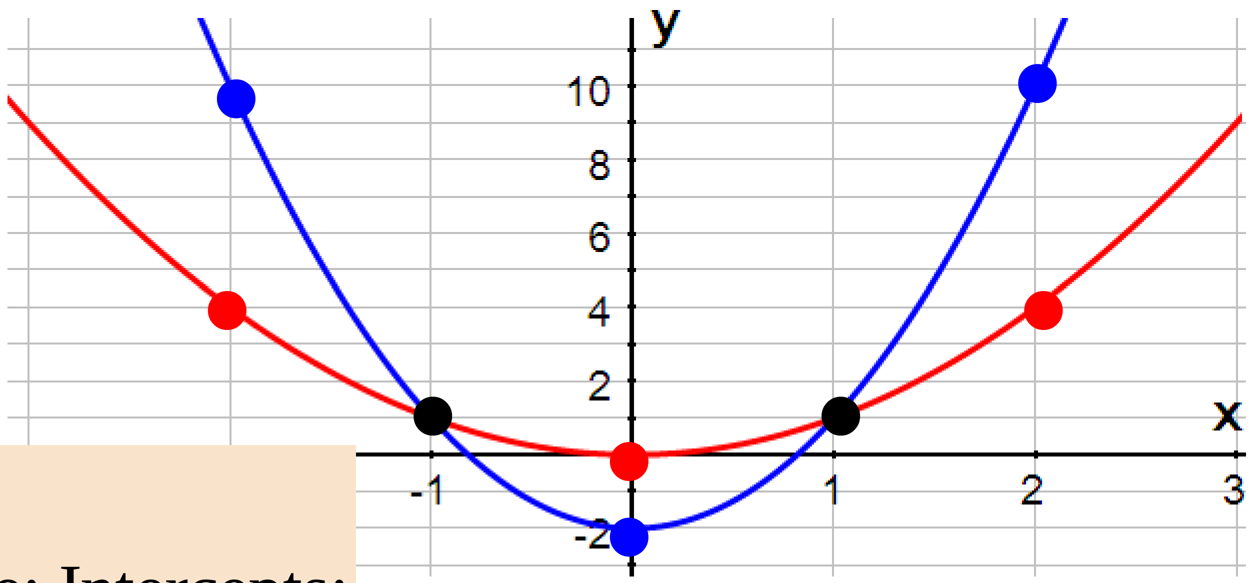
- Points on basic functions can be transformed to generate points on the transformed functions.
- We can utilize the effect of parameters on points on the basic function to sketch transformed function.
- Transformation can have an impact on the characteristics of the basic functions.
- Should take note of changes to the following: Domain and Range; Intercepts with axes; Turning points; Minima and Maxima; Asymptotes; Shape and Symmetry as well as Periodicity and Amplitude.

Function defined by $y = ax^2 + q$

Consider the example $y = 3x^2 - 2.$

Consider transformations of points on $y = x^2$:

$(-2;4)$	$\rightarrow$	$(-2;10)$
$(-1;1)$	$\rightarrow$	$(-1;1)$
$(0;0)$	$\rightarrow$	$(0;-2)$
$(1;1)$	$\rightarrow$	$(1;1)$
$(2;4)$	$\rightarrow$	$(2;10)$



Impact on :

Domain and Range; Intercepts;

Turning Point; Symmetry

Tutorial 3: Transformed Parabola

Given the functions defined by $y = x^2$ and $y = 2\left(x-3\right)^2 + 4$

(1) Sketch basic function defined by $y = 2x^2$

(2) Make use of transformations and sketch $y = 2\left(x-3\right)^2 + 4$

(3) Discuss changes to:

(a) Domain and Range

(b) Intercepts

(c) Turning point

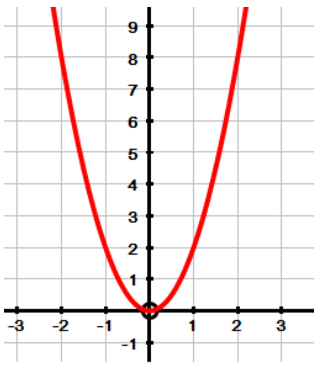
(d) Symmetry

PAUSE DVD

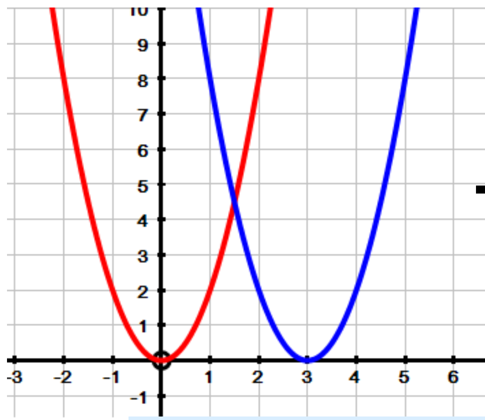
- Do Tutorial 3
- Then View Solutions

Tutorial 3 Parabola: Suggested Solution

$y = 2x^2$



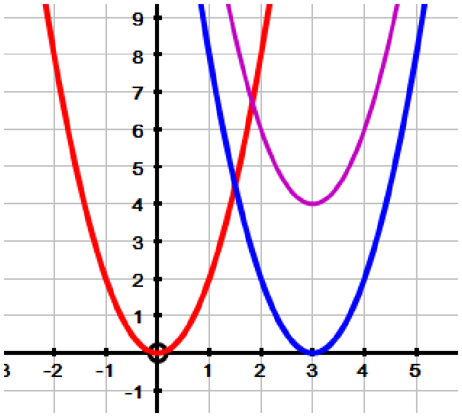
$y = 2(x-3)^2$



Horizontal shifts

of 3 units to right

$y = 2(x-3)^2 + 4$



Vertical shifts

of 4 units upwards

$D = \mathbb{R}$ and $R = [4;\infty)$

Imaginary roots.

$y-$intercept where $x = 0$

$\therefore$ At $(0;22)$

Turning point:

Effected by parameters -3 and 4. $\therefore$ At $(3;4)$

Axis of Symmetry: Effected by parameter -3

Defined by $x = 3$

Function defined by $y = \frac{a}{x+p} + q$ where $a, p, q \in \mathbb{Z}$

- Asymptotes:
  - Vertical Asymptote defined by $x = -p$
  - Horizontal Asymptote defined by $y = q$

- Points $(x; y) \xrightarrow{\text{Transformed to}} (x - p; ay + q)$

- $x-$intercept where $y = 0$ and $y-$intercept where $x = 0$

- Symmetry between two branches changes from $y = -x$ to $y = -a(x+p) + q$

Tutorial 4: Transformed Hyperbolic Function

A function is defined by $y = \dfrac{1}{x-4} + 5$:

(1) Write down the defining equation of the horizontal asymptote;

(2) Write down the defining equation of the vertical asymptote;

(3) Determine the $x-$intercept(s);

(4) Determine the $y-$intercept;

(5) Write down the domain and range;

(6) Determine the defining equations of the lines of symmetry between two branches and an individual branch;

(7) Sketch the transformed function.

PAUSE DVD

- Do Tutorial 4
- Then View Solutions

Tutorial 4 Problems 1 to 4: Suggested Solutions

Function is defined by $y = \dfrac{1}{x-4} + 5$

(1) Horizontal asymptote is defined by $y = 5$

(2) Vertical asymptote is defined by $x = 4$

(3) $x-$intercept is where $y = 0$

This is where $0 = \dfrac{1}{x-4} + 5 \Rightarrow 0 = 1 + 5\left(x-4\right)$

Thus when $5x - 20 + 1 = 0 \Rightarrow x = \dfrac{19}{5} = 3{,}8$

(4) $y-$intercept is where $x = 0 \Rightarrow y = \dfrac{1}{-4} + 5 = 4\dfrac{3}{4} = 4{,}75$

Tutorial 4 Problem 5 to 6: Suggested Solutions

A function is defined by $y = \frac{1}{x-4} + 5$:

(5) $D = \{x : x \neq 4\}$ and $R = \{y : y \neq 5\}$

(6) Line of symmetry between two branches changes from

$y = -x$ to $y = -(x-4)+5$ or $y = -x+9$

Line of symmetry for an individual branch changes from

$y = x$ to $y = (x-4)+5$ or $y = x+1$

Tutorial 4 Problem 7: Suggested Solution

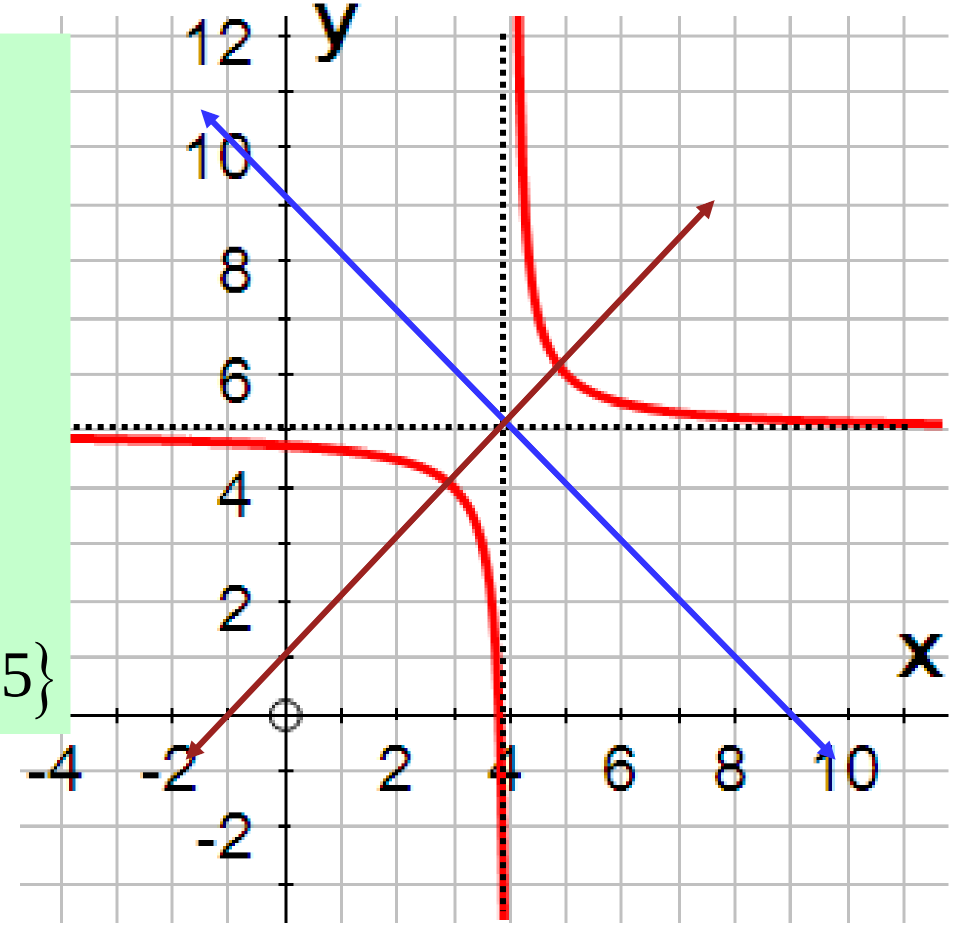
(7) Sketch defined by $y = \dfrac{1}{x-4} + 5$

Check List :

- Asymptotes: $y = 5$ and $x = 4$
- Symmetry Lines:

  $y = -x + 9$ and $y = x + 1$
- Intercepts at:

  $(3,8;0)$ and $(0;4,75)$
- $D = \{x : x \neq 4\} \wedge R = \{y : y \neq 5\}$



Tutorial 5: Transformed Exponential Function

The basic function defined by $y = 2^x$ is transformed

to a function defined by $y = 3 \times 2^{x-2} - 1$.

For the transformed function:

(1) Write down the defining equation of the horizontal asymptote;

(2) Determine a possible $x-$intercept;

(3) Determine a possible $y-$intercept;

(4) Write down the domain and range;

(5) Sketch the transformed function.

PAUSE DVD

- Do Tutorial 5
- Then View Solutions

Tutorial 5 Problems 1 to 3: Suggested Solutions

Function is defined by $y = 3\times 2^{x-2} - 1.$

(1) Horizontal asymptote is defined by $y = -1$

(2) $x-$intercept where $y = 0$ or where $3\times 2^{x-2} - 1 = 0$

$$\Rightarrow (x-2)\log 2 = \log\left(\frac{1}{3}\right) = -\log 3$$

$$\Rightarrow x = \frac{-\log 3}{\log 2} + 2 \approx 0,415$$

(3) $y-$intercept where $x = 0$

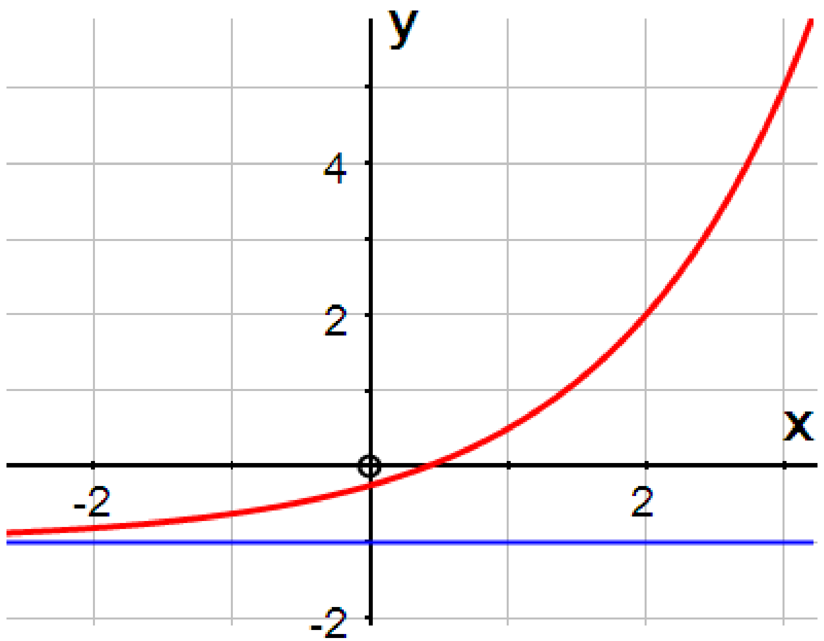
$$\Rightarrow y = 3\times 2^{-2} - 1 = \frac{3}{4} - 1 = -\frac{1}{4}$$

Tutorial 5 Problems 4 and 5: Suggested Solutions

Transformed function defined by $y = 3 \times 2^{x-2} - 1$.

(4) $D = \mathbb{R}$ and $R = (-1; \infty)$

(5) Sketch the transformed function.



Consider the following transformed points:

Points on $y = 2^x$	Points on $y = 3 \times 2^{x-2} - 1$
$\left(-2; \frac{1}{4}\right)$	$\left(0; -\frac{1}{4}\right)$
$\left(-1; \frac{1}{2}\right)$	$\left(1; \frac{1}{2}\right)$
$(0;1)$	$(2;2)$
$(1;2)$	$(3;5)$
$(2;4)$	$(4;11)$

Check List :

Horizontal Asymptote at $y = -1$

Intercepts at $(0,415;0)$ and $(0;-0,25)$

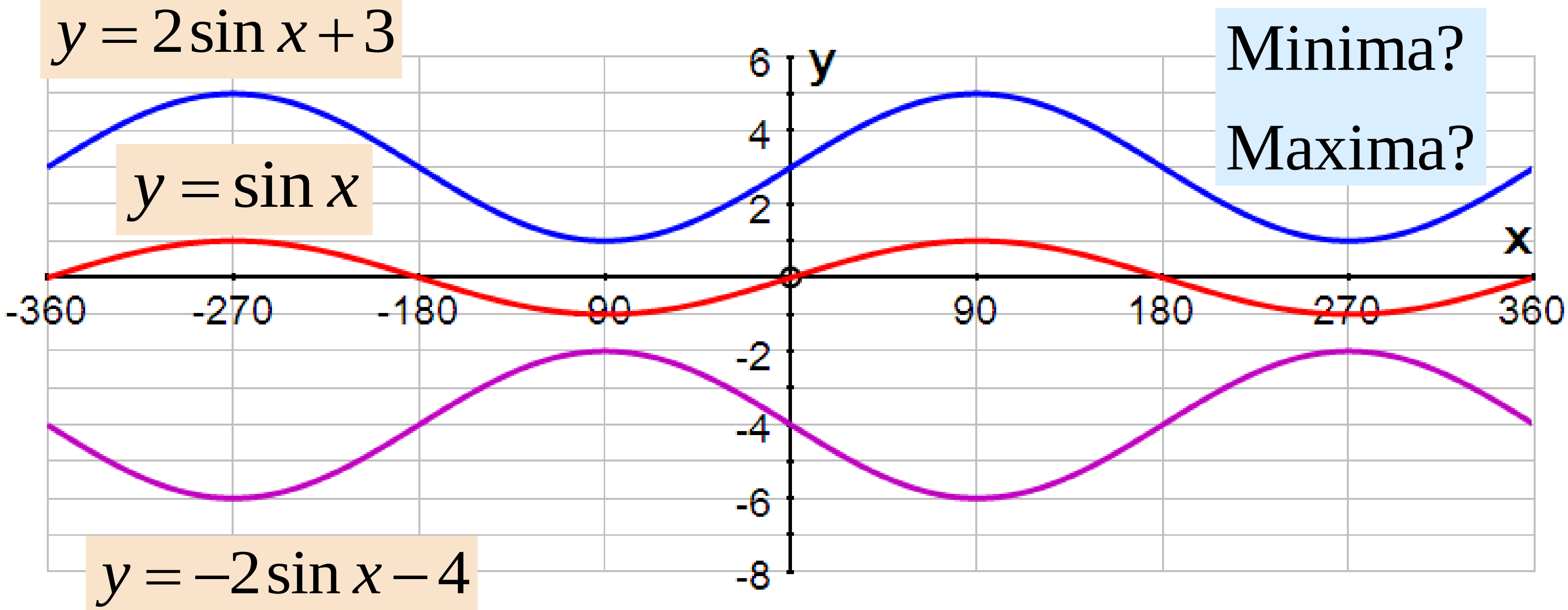
Function defined by $y = a\sin x + q$

$y = 2\sin x + 3$

- Vertical stretch by factor 2 followed by a vertical shift by 3 units upward.

$R = [1;5]$

Amplitude is $(5-1) \div 2 = 2$



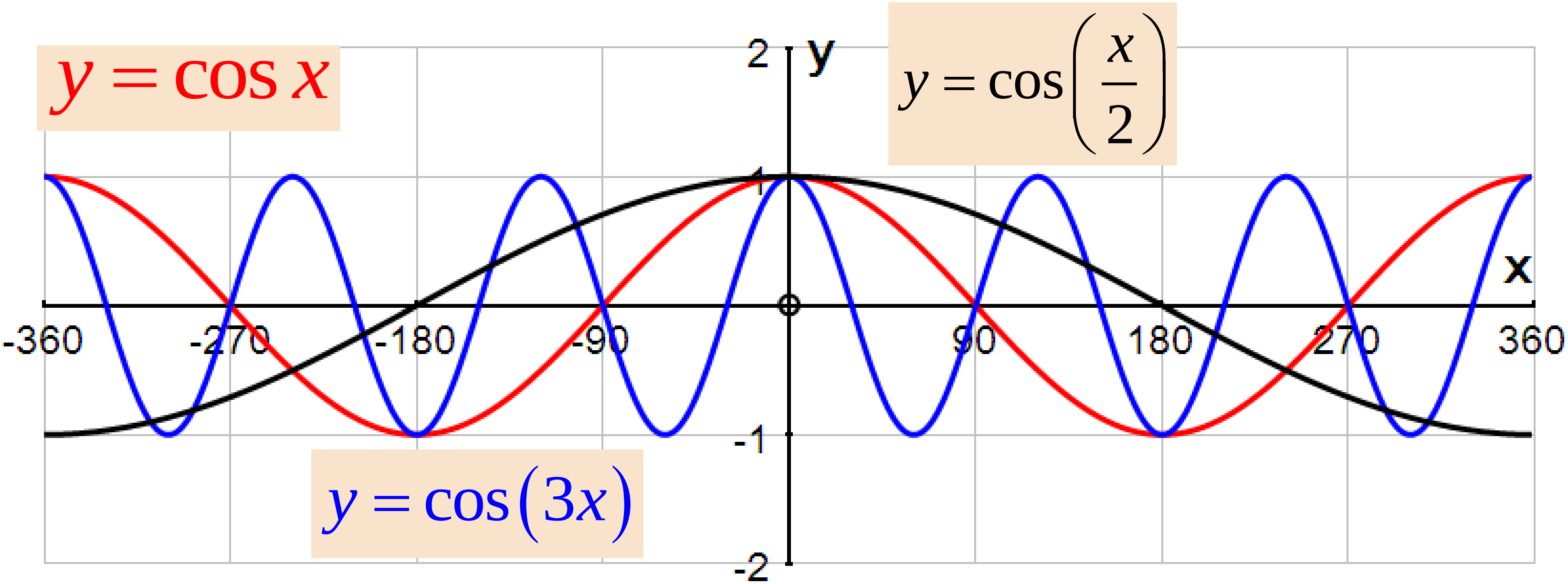
$y = -2\sin x - 4$

- Vertical stretch by factor 2; reflection in $x-$axis followed by a vertical shift by 4 units downward.

$R = [-6;-2]$

Amplitude is 2

Function defined by $y = \cos(kx)$



$y = \cos(3x)$

Period changed from 360° to 120°.

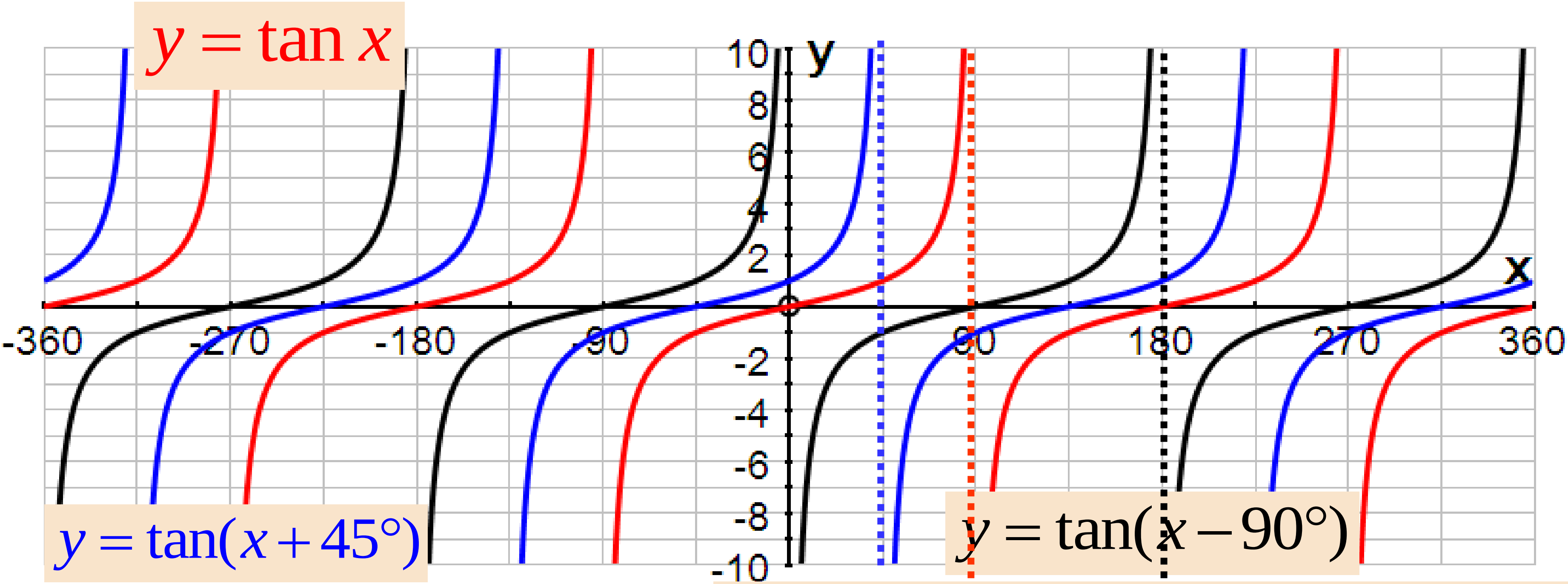
- Horizontal Compression by a factor 3.

$y = \cos\left(\frac{x}{2}\right)$

Period changed from 360° to 720°.

- Horizontal Stretch by a factor 2.

Function defined by $y = \tan(x + p)$



$y = \tan(x + 45°)$

- Horizontal Shift by 45° to the left.

Asymptotes changes from:

$x = 90° + 180° \cdot n$ to $x = 45° + 180° \cdot n;\ n \in \mathbb{Z}$

$y = \tan(x - 90°)$

- Horizontal Shift by 90° to the right.

Asymptotes changes from:

$x = 90° + 180° \cdot n$ to $x = 180° + 180° \cdot n;\ n \in \mathbb{Z}$

Tutorial 6: Transformed Trigonometric Functions

- Sketch the given transformed trigonometric functions defined below.
- Discuss changes in the characteristic: Domain; Range; Periodicity; Amplitude; Minima; Maxima and Asymptotes.

(1) $y = 3\cos x + 2$ if $x \in [0°; 360°]$;

(2) $y = \sin(3x + 90°)$ if $x \in [0°; 360°]$ and

(3) $y = \tan[2(x + 45°)]$ if $x \in [0°; 270°]$.

PAUSE DVD

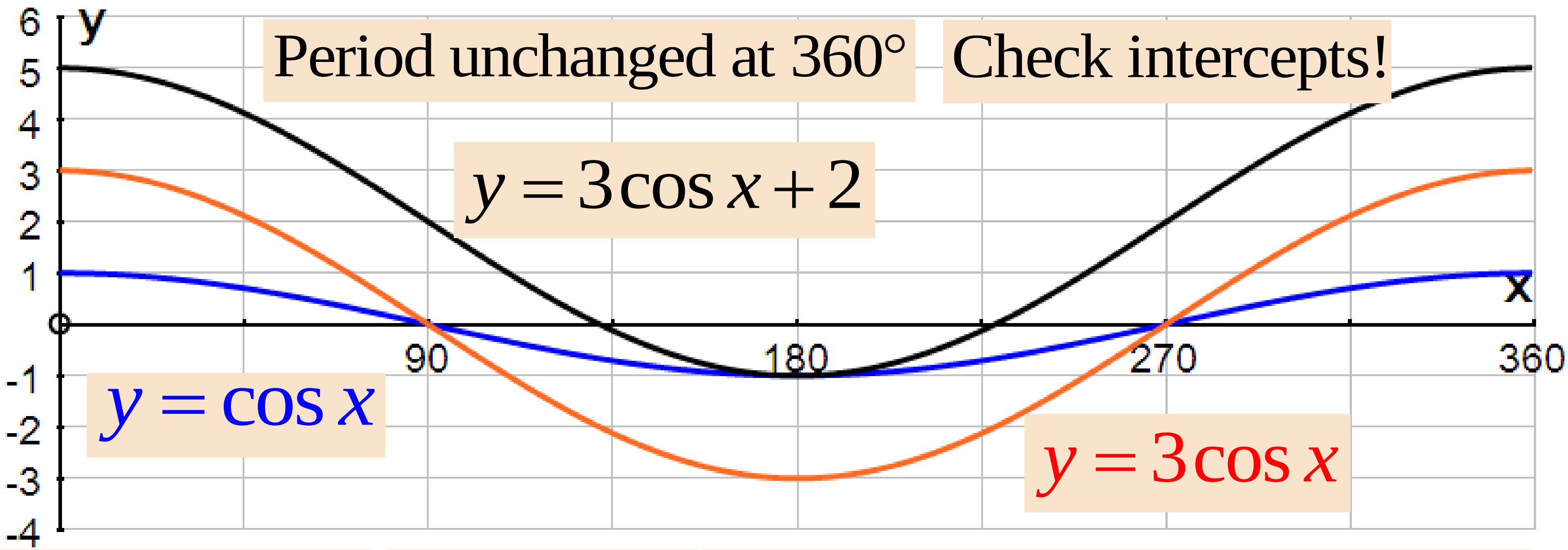
- Do Tutorial 6
- Then View Solutions

Tutorial 6 Problem 1: Suggested Solution

$$(1)\ y = 3\cos x + 2 \quad \text{if} \quad x \in [0°; 360°]$$

Only one possible sequencing:

- $y = \cos x \xrightarrow[\text{by a factor 3}]{\text{Vertical Stretch}} y = 3\cos x \xrightarrow[\text{by 2 units upward}]{\text{Vertical Shift}} y = 3\cos x + 2$



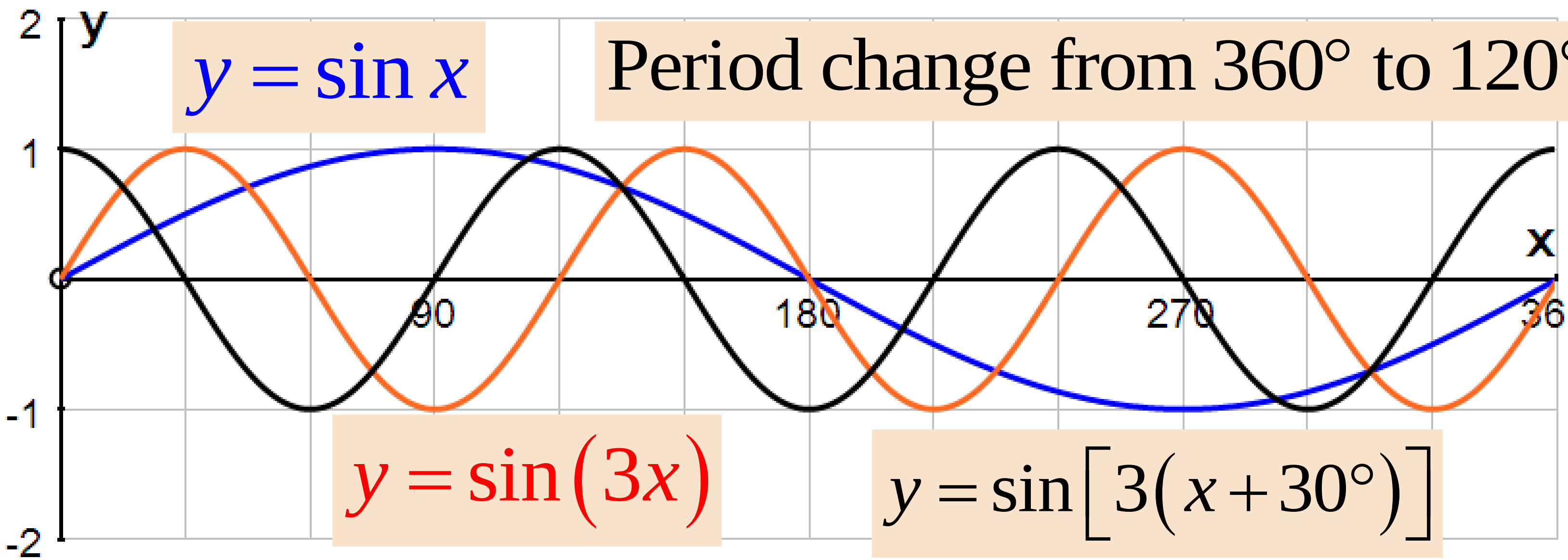
$D = [0°; 360°]$ $R = [-1; 5]$ Amplitude: $[5-(-1)] \div 2 = 3$

Tutorial 6 Problem 2: Suggested Solution

(2) $y=\sin(3x+90°)=\sin[3(x+30°)]$ if $x\in[0°;360°]$.

Only one possible sequencing:

- $y=\sin x \xrightarrow[\text{by a factor 3}]{\text{Horizontal Compression}} y=\sin(3x) \xrightarrow[\text{by 30° to left}]{\text{Horizontall Shift}} y=\sin[3(x+30°)]$



$y=\sin x$

Period change from 360° to 120°

$y=\sin(3x)$

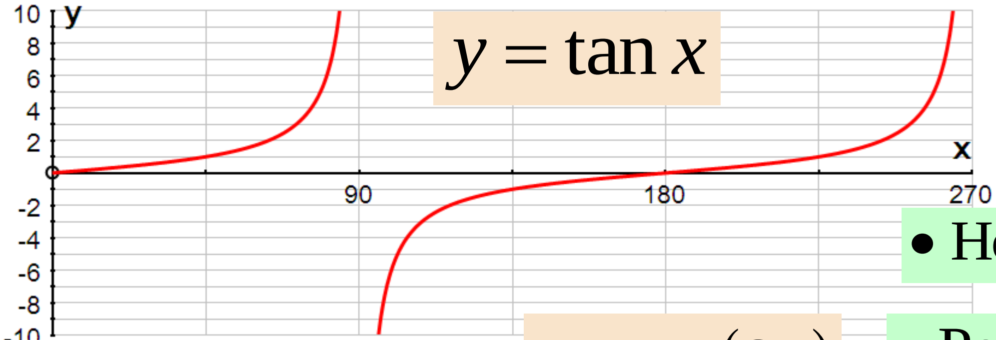
$y=\sin[3(x+30°)]$

$D=[0°;360°]$ $R=[-1;1]$ Amplitude is 1 Check intercepts!

Tutorial 6 Problem 3: Suggested Solution

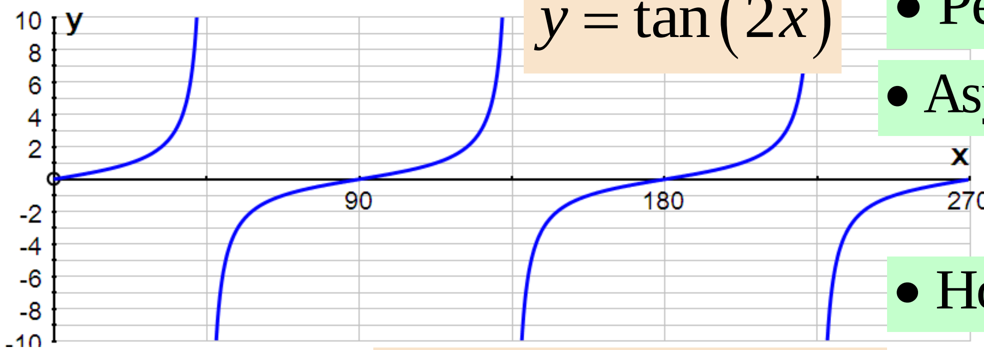
(3) $y = \tan\left[2\left(x + 45°\right)\right]$ if $x \in \left[0°; 270°\right]$.

$y = \tan x$



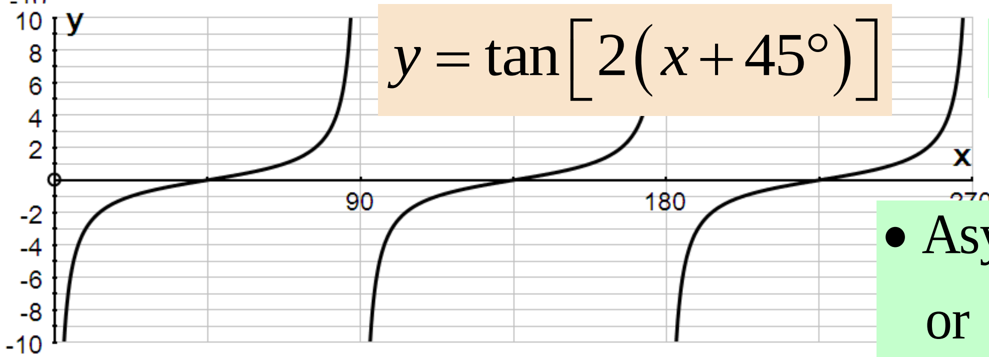
- Horizontal Compression by factor 2.
- Period changes from 180° to 90°.
- Asymptotes at $x = 45° + 90° \cdot n$; $n \in \mathbb{Z}$

$y = \tan\left(2x\right)$



- Horizontal Shift by 45° to the left.

$y = \tan\left[2\left(x + 45°\right)\right]$



- Period unchanged at 90°.
- Asymptotes at $x = 0° + 90° \cdot n$; $n \in \mathbb{Z}$

  or $x = 90° \cdot n$; $n \in \mathbb{Z}$

End of the DVD on Functions and their Graphs

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in the DVD.
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!