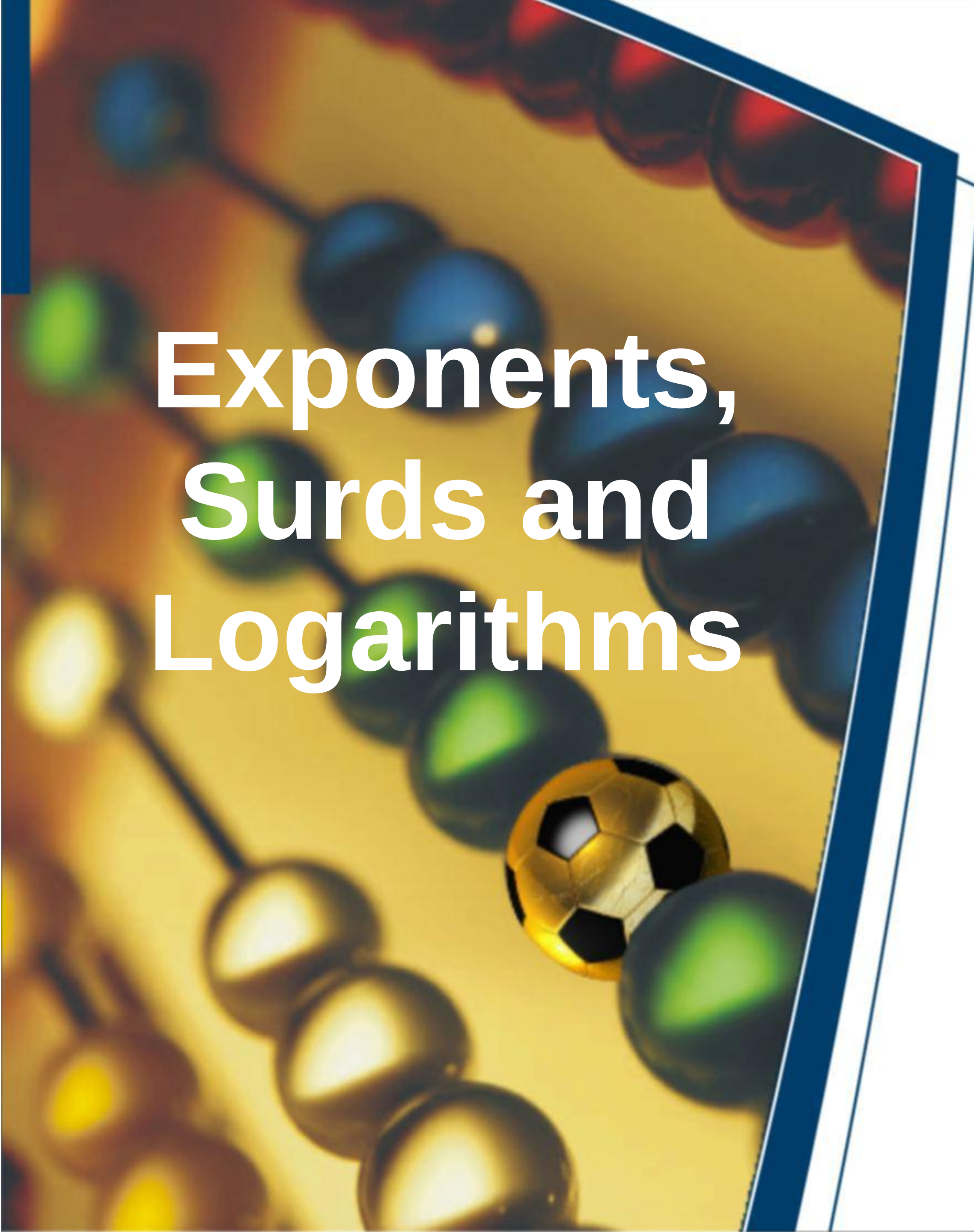
Exponents, Surds and Logarithms



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Outcomes for this DVD

In this DVD you will:

- Revise the exponential notation and review the index laws.

  LESSON 1.

- Simplify expressions involving rational exponents.

  LESSON 2.

- Simplify expressions involving surds.

  LESSON 3.

- Revise the logarithmic notation and logarithm laws.

  LESSON 4.

Lesson 1

The Exponential Notation and Index Laws

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The Exponential Notation

$2^4 = 2 \times 2 \times 2 \times 2$ or 2^4 is the product of 4 factors of 2.

We know that :

$$a^n = a \times a \times a \times \ldots \times a \quad (\text{to } n \text{ factors of } a,\ n \in \mathbb{N},\ a \in \mathbb{R})$$

$$r = a^n$$

Exponent

Power

Base

What if exponents are not positive integers?

If variable bases are non-zero and n is a positive integer then:

1. $a^0 = 1 \quad (0^0 \text{ is undefined})$

2. $a^{-n} = \dfrac{1}{a^n}$

Examples

1) $\left(-3a^2\right)^0 = 1$

2) $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

Index Law 1 (Multiplication)

$$\text{Law 1: } a^n \times a^m = a^{n+m}$$

Examples

1) $a^4 \times a^5 = a^9$

2) $2^{n+1}.2 = 2^{n+2}$

3) $-(2)^2.(-2) = 2^3 = 8$

4) $(-3)^3.(-3)^2 = (-3)^5 = -243$

Index Law 2 (Division)

$$\text{Law 2: } a^n \div a^m = a^{n-m}$$

Examples

1) $\dfrac{a^5}{a^3} = a^{5-3} = a^2$

2) $\dfrac{5^6}{5^8} = 5^{6-8} = 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

3) $\dfrac{2ab^2}{10a^2b} = \dfrac{a^{-1}b}{5} = \dfrac{b}{5a}$

Index Law 3 (Exponentiation)

$$\text{Law 3:} \quad \left(a^{n}\right)^{m} = a^{nm}$$

Examples

1) $\left(a^{2}\right)^{3} = a^{2\times 3} = a^{6}$

2) $\left(x^{5}\right)^{2} = x^{5\times 2} = x^{10}$

Index Law 4

$$\text{Law 4: } (ab)^m = a^m b^m$$

Examples

1) $\left(2^3 a^4\right)^2 = 2^{3\times 2} a^{4\times 2} = 2^6 a^8 = 64a^8$

2) $\left(2^3 \times 3^2\right)^3 = 2^{3\times 3} \times 3^{2\times 3} = 2^9 \times 3^6$

Index Law 5

$$\text{Law 5:} \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Examples

1) $$\left(\frac{2^3}{3^4}\right)^3 = \frac{2^{3\times3}}{3^{4\times3}} = \frac{2^9}{3^{12}}$$

2) $$\left(\frac{2}{x}\right)^2 = \frac{2^2}{x^2} = \frac{4}{x^2}$$

Example 1: Applying Exponent Laws

- **Simplification using the exponent laws.**

1) $$\frac{6^{6x}.9^{3x}}{18^{4x}.\left(\frac{1}{4}\right)^{2-x}} = \frac{(2.3)^{6x}.\left(3^2\right)^{3x}}{\left(2.3^2\right)^{4x}.\left(2^{-2}\right)^{2-x}}$$

$$= \frac{2^{6x}.3^{6x}.3^{6x}}{2^{4x}.3^{8x}.2^{-4+2x}}$$

$$= 2^{6x-4x+4-2x}.3^{6x+6x-8x}$$

$$= 2^4.3^{4x} \; or \; 16 \times 3^{4x}$$

Example 2: Applying Exponent Laws

- **Simplification using the exponent laws.**

$$2)\quad \frac{2^{n+1}}{\left(2^{n}\right)^{n-1}} \div \frac{4^{n+1}}{\left(2^{n-1}\right)^{n+1}}$$

Multiply with the reciprocal

$$= \frac{2^{n+1}}{2^{n^2-n}} \times \frac{2^{n^2-1}}{2^{2n+2}}$$

$$= 2^{n+1+n^2-1-n^2+n-2n-2} = 2^{-2} = \frac{1}{4}$$

Tutorial 1: Simplify Expressions using the Exponent Laws

Simplify the following expressions:

1) $$\frac{2^{3n+2}.8^{n-3}}{4^{3n-2}.\left(\frac{1}{4}\right)^2}$$

2) $$\frac{12^{x-2}\times 2^{x+2}}{8^x\times 3^{x-4}}$$

3) $$\frac{18^x\left(2.3^{1-x}\right)^2}{2^{x-1}}$$

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- Do Tutorial 1
- Then View Solutions

Tutorial 1 Problem 1: Suggested Solution

1) $$\frac{2^{3n+2}.8^{n-3}}{4^{3n-2}.\left(\frac{1}{4}\right)^{2}}$$

$$=\frac{(2)^{3n+2}.(2)^{3n-9}}{(2)^{6n-4}.(2)^{-4}}$$

$$=2^{3n+2+3n-9-6n+4+4}=2$$

Tutorial 1 Problem 2: Suggested Solution

$$2)\quad \frac{12^{x-2}\times 2^{x+2}}{8^{x}\times 3^{x-4}} = \frac{\left(3\times 2^{2}\right)^{x-2}\times 2^{x+2}}{\left(2^{3}\right)^{x}\times 3^{x-4}}$$

$$= \frac{2^{2x-4}.3^{x-2}.2^{x+2}}{2^{3x}.3^{x-4}}$$

$$= 2^{2x-4+x+2-3x}.3^{x-2-x+4}$$

$$= 2^{-2}.3^{2} = \frac{9}{4}$$

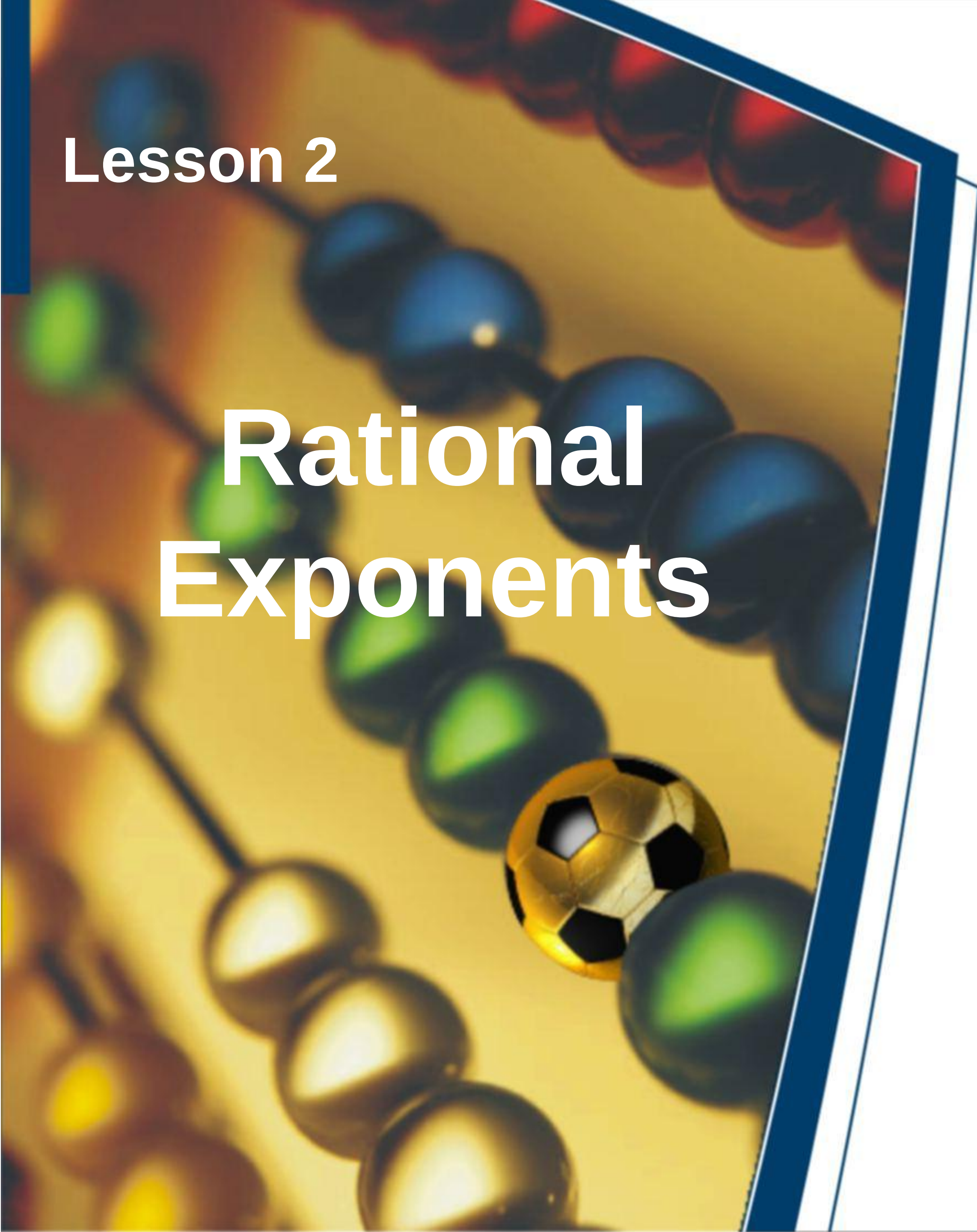
Tutorial 1 Problem 3: Suggested Solution

3) $$\frac{18^{x}\left(2.3^{1-x}\right)^{2}}{2^{x-1}}$$

$$=\frac{\left(2^{x}\cdot 3^{2x}\right)\left(2^{2}\cdot 3^{2-2x}\right)}{2^{x-1}}$$

$$=2^{x+2-x+1}.3^{2x+2-2x}$$

$$=2^{3}\times 3^{2}=72$$



Lesson 2

Rational Exponents

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What is a rational exponent?

Rational Number is a real number which can be written in the form

$$\frac{m}{n} \text{ where } m, n \in \mathbb{Z} \text{ and } n \neq 0$$

We will in this part consider powers where the exponent is a rational number.

$\therefore$ We will consider expressions like $a^{\frac{m}{n}}$

Equivalent Notations

$$a^{\frac{1}{n}} = \sqrt[n]{a}\ (n \in \mathbb{N}, n \geq 2,)$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}}\ \text{(From right to left)}$$

$$4^{\frac{1}{5}} = \sqrt[5]{4}\ \text{(From left to right)}$$

Equivalent Notations for negative rational exponents

$$a^{-\frac{1}{n}} = a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}} \ (n \in \mathbb{N}, n \geq 2,)$$

$$5^{-\frac{1}{3}} = \sqrt[3]{5^{-1}} = \sqrt[3]{\frac{1}{5}} \text{ (From left to right)}$$

$$\sqrt[5]{\frac{1}{4}} = \sqrt[5]{4^{-1}} = 4^{-\frac{1}{5}} \text{ (From right to left)}$$

Another Equivalent Notation

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = r \ (a > 0, n \in \mathbb{N}; m, n \geq 2)$$

$$2^{\frac{2}{3}} = \sqrt[3]{2^2} \quad \text{(From left to right)}$$

$$\sqrt[4]{3^{-5}} = 3^{\frac{-5}{4}} \quad \text{(From right to left)}$$

Simplification (Without a calculator) of single term exponential expressions

- **Factorize all bases into prime factors.**
- **Apply the exponential laws.**

$$1)\ 81^{\frac{1}{4}} \times 27^{-\frac{2}{3}}$$

$$= \left(3^4\right)^{\frac{1}{4}} \times \left(3^3\right)^{-\frac{2}{3}} = 3^1 \times 3^{-2} = 3^{-1} = \frac{1}{3}$$

$$2)\ 125^{-\frac{1}{6}} \times 25^{\frac{1}{4}}$$

$$= \left(5^3\right)^{-\frac{1}{6}} \times \left(5^2\right)^{\frac{1}{4}} = 5^{-\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^0 = 1$$

Example 1: Simplification of polynomial exponential expressions

- **Without variables – simplify each term and add.**

$$1)\ \ 125^{2/3} + 16^{3/4} - 8^{-2/3}$$

$$= \left(5^3\right)^{2/3} + \left(2^4\right)^{3/4} - \left(2^3\right)^{-2/3}$$

$$= 5^2 + 2^3 - 2^{-2}$$

$$= 25 + 8 - 1/4 = 32\,3/4$$

Example 2: Simplification of polynomial exponential expressions

- **Without variables – simplify each term and add.**
- **With variables – factorize.**

2) $$\frac{9^{x/2} - 3^{x-1}}{3^{x+2}}$$

$$= \frac{\left(3^2\right)^{x/2} - 3^{-1}.3^x}{3^x.3^2}$$

$$= \frac{3^x - 3^x.\,{}^{1}\!/\!{}_{3}}{3^x.9} = \frac{3^x\left(1 - {}^{1}\!/\!{}_{3}\right)}{3^x.9} = \frac{{}^{2}\!/\!{}_{3}}{9} = \frac{2}{3} \times \frac{1}{9} = \frac{2}{27}$$

Example 3: Simplification of polynomial exponential expressions

- **Without variables – simplify each term and add.**
- **With variables – factorize.**

3) $$\frac{3.2^{m}-4.2^{m+2}}{2^{m}-2^{m+1}}$$

$$=\frac{2^{m}\left(3-16\right)}{2^{m}\left(1-2\right)}=\frac{-13}{-1}=13$$

Example 4: Simplification of polynomial exponential expressions

- **Without variables – simplify each term and add.**
- **With variables – factorize.**

4) $$\frac{2^{2x}-2^{x}-6}{2^{x}+2}$$

$$=\frac{\left(2^{x}+2\right)\left(2^{x}-3\right)}{\left(2^{x}+2\right)}$$

$$=2^{x}-3$$

<u>Note:</u> Replace 2^{x} with a

$$\left(2^{x}\right)^{2}-2^{x}-6$$

$$=a^{2}-a-6$$

$$=\left(a+2\right)\left(a-3\right)$$

$$=\left(2^{x}+2\right)\left(2^{x}-3\right)$$

Example 5: Simplification of polynomial exponential expressions

- **Without variables – simplify each term and add.**
- **With variables – factorize.**

5) $$\left[\frac{2^{x+1}+6.2^{x-1}}{5.4^{x}}\right]^{1/x}=\left[\frac{2^{x}(2+3)}{5.2^{2x}}\right]^{1/x}$$

$$=\left[\frac{5.2^{x}}{5.2^{2x}}\right]^{1/x}=\left(2^{-x}\right)^{1/x}=2^{-1}=\frac{1}{2}$$

Tutorial 2: Working with exponents

1) Simplify without calculators:

(a) $\left(\frac{4}{9}\right)^{3/2} + (0,125)^{-2/3}$

(b) $\frac{5^{-1}}{3^{2}} \times \frac{4^{0}}{27^{2/3}} \div 15^{-2}$

2) Simplify:

(a) $\frac{12^{x-2} \times 2^{x+2}}{8^{x} \times 3^{x-4}}$

(b) $\frac{4^{x} - 2^{2x+1}}{\left(2^{x}\right)^{2} - 2^{x+3} \times 2^{x}}$

(c) $\frac{5^{a-2}.2^{a+2}}{10^{a} - 10^{a-1}.2}$

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Tutorial 2 Problem 1(a): Suggested solution

1a) Simplify, without using a calculator:

$$\left(\frac{4}{9}\right)^{3/2} + (0{,}125)^{-2/3}$$

$$= \left(\left(\frac{2}{3}\right)^2\right)^{3/2} + \left(\left(\frac{1}{2}\right)^3\right)^{-2/3}$$

$$= \left(\frac{2}{3}\right)^3 + \left(\frac{1}{2}\right)^{-2} = \frac{8}{27} + 2^2 = 4\frac{8}{27}$$

Tutorial 2 Problem 1(b): Suggested solution

1(b) Simplify (No calculators):

$$\frac{5^{-1}}{3^2} \times \frac{4^0}{27^{2/3}} \div 15^{-2}$$

$$= \frac{1}{3^2.5} \times \frac{1}{\left(3^3\right)^{2/3}} \times \frac{1}{\left(3 \times 5\right)^{-2}}$$

$$= \frac{1}{3^2.5} \times \frac{1}{3^2} \times \frac{1}{3^{-2}5^{-2}} = \frac{1}{3^2.5^{-1}} = \frac{5}{9}$$

Tutorial 2 Problem 2(a): Suggested solution

2a) Simplify:

$$\frac{12^{x-2} \times 2^{x+2}}{8^{x} \times 3^{x-4}} = \frac{\left(2^{2}.3\right)^{x-2} \times 2^{x+2}}{2^{3x} \times 3^{x-4}}$$

$$= \frac{2^{2x-4}.3^{x-2}.2^{x+2}}{2^{3x}.3^{x-4}}$$

$$= 2^{2x-4+x+2-3x}.3^{x-2-x+4}$$

$$= 2^{-2}.3^{2} = \frac{9}{4}$$

Tutorial 2 Problem 2(b): Suggested solution

2b) Simplify:

$$\frac{4^x - 2^{2x+1}}{\left(2^x\right)^2 - 2^{x+3} \times 2^x}$$

$$= \frac{2^{2x} - 2^{2x+1}}{2^{2x} - 2^{2x+3}}$$

$$= \frac{2^{2x}\left(1-2\right)}{2^{2x}\left(1-8\right)} = \frac{-1}{-7} = \frac{1}{7}$$

Tutorial 2 Problem 2(c): Suggested solution

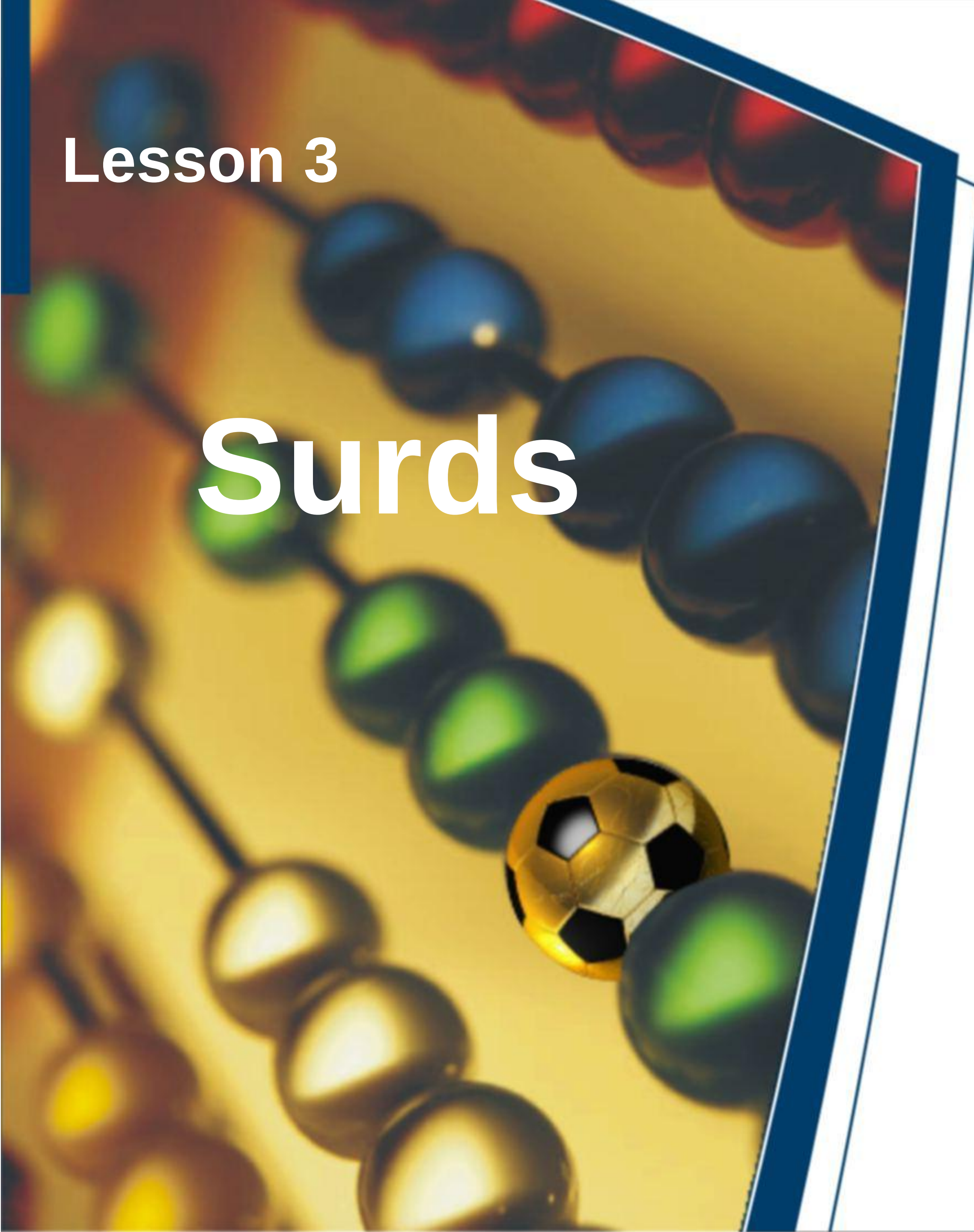
2(c) Simplify: $\dfrac{5^{a-2}.2^{a+2}}{10^a - 10^{a-1}.2}$

$$= \frac{5^a \cdot 5^{-2}.2^a.2^2}{(5.2)^a - (5.2)^{a-1}.2} = \frac{5^a 5^{-2}.2^a.2^2}{5^a.2^a - 5^{a-1}.2^{a-1}.2}$$

$$= \frac{5^a 5^{-2}.2^a.2^2}{5^a.2^a(1-5^{-1}.2^{-1}.2)} = \frac{5^{-2}2^2}{1-1/5} = \frac{\frac{4}{25}}{\frac{4}{5}} = \frac{4}{25} \times \frac{5}{4} = \frac{1}{5}$$

Lesson 3

Surds



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Definition of Surds

We Define $\longrightarrow$

If $\sqrt[n]{a} = x$ then $x^n = a$,

where $n \in \mathbb{N}$ and $a \in \mathbb{R}$.

NOTE

If n is even, we must have $a \geq 0$ and $x \geq 0$.

Thus $\sqrt[n]{a} = a^{1/n}$

Examples directly from the Definition

1) $\sqrt[3]{8} = 8^{1/3} = \left(2^3\right)^{1/3} = 2$

Use definition to check!

2) $\sqrt[5]{-32} = (-32)^{1/5} = \left[(-2)^5\right]^{1/5} = -2$

3) $\sqrt[4]{81} = 81^{1/4} = \left(3^4\right)^{1/4} = 3$

Note: $\sqrt[2]{a} = \sqrt{a}$

4) $\sqrt[2]{-16} = \sqrt{-16} = (-16)^{1/2} \neq \left[(-4)^2\right]^{1/2}$

$\therefore \sqrt{-16}$ has no meaning (Imaginary numbers)

Multiplication Property for Surds

Property 1: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

Applications of the Multiplication Property

1) $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (Left to right)

2) $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ (Right to left)

Composition Property for Surds

Property 2: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Applications of the Composition Property

3) $\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$ (From left to right)

4) $\sqrt[6]{16} = \sqrt[3]{\sqrt[2]{16}} = \sqrt[3]{4}$ (From right to left)

Exponent Property for Surds

Property 3: $\left(\sqrt[m]{a}\right)^n = \sqrt[m]{a^n}$

Application of the Exponent Property

5) $\left(\sqrt[4]{2a}\right)^3 = \sqrt[4]{(2a)^3} = \sqrt[4]{8a^3}$

Division Property for Surds

Property 4: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Applications of the Division Property

6) $\dfrac{\sqrt{2}}{\sqrt{6}} = \sqrt{\dfrac{1}{3}}$ (From left to right)

7) $\sqrt{\dfrac{9}{4}} = \dfrac{\sqrt{9}}{\sqrt{4}} = \dfrac{3}{2}$ (From right to left)

Example 1: Simplification of surd expressions without using a calculator

1) $2\sqrt{8} - 4\sqrt{32} + 3\sqrt{50}$

$$= 2\sqrt{4}\sqrt{2} - 4\sqrt{16}\sqrt{2} + 3\sqrt{25}\sqrt{2}$$

$$= 4\sqrt{2} - 16\sqrt{2} + 15\sqrt{2}$$

$$= 3\sqrt{2}$$

Example 2: Simplification of surd expressions without using a calculator

2) $\sqrt{50}\left(\sqrt{18}+\sqrt{32}\right)$

$$=5\sqrt{2}\left(3\sqrt{2}+4\sqrt{2}\right)$$

$$=5\sqrt{2}\left(7\sqrt{2}\right)$$

$$=35\times 2=70$$

Note: $\sqrt{a}\times\sqrt{a}=a$ also $\sqrt[3]{a}\times\sqrt[3]{a}\times\sqrt[3]{a}=a\cdots$

Example 3: Simplification of surd expressions without using a calculator

3) $\left(2-\sqrt{3}\right)^2+\left(2+\sqrt{3}\right)^2$

$=4-4\sqrt{3}+3+4+4\sqrt{3}+3$

$=14$

Example 4: Simplification of surd expressions without using a calculator

4) $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{8}}$

$$= \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2+1}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{4}$$

Moving the surd from the denominator to the numerator

Example 5: Simplification of surd expressions without using a calculator

5) $\dfrac{1}{\sqrt{3}+2} - \dfrac{1}{\sqrt{3}-2}$

$$= \frac{\left(\sqrt{3}-2\right)-\left(\sqrt{3}+2\right)}{\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)}$$

$$= \frac{-4}{3-4} = 4$$

Example 6: Simplification of surd expressions without using a calculator

6) $\left(\dfrac{1}{\sqrt{8}+\sqrt{2}}-\dfrac{1}{\sqrt{8}+3\sqrt{2}}\right)^{-1}$

$$=\left(\frac{1}{3\sqrt{2}}-\frac{1}{5\sqrt{2}}\right)^{-1}$$

$$=\left(\frac{5-3}{15\sqrt{2}}\right)^{-1}=\frac{15\sqrt{2}}{2}$$

Example 7: Simplification of surd expressions without using a calculator

$$7)\quad \frac{5\sqrt{11}.\sqrt{12}}{\sqrt{45}.\sqrt{33}}$$

$$= \frac{5\sqrt{11}.2\sqrt{3}}{3\sqrt{5}.\sqrt{3}\sqrt{11}}$$

$$= \frac{10}{3\sqrt{5}} = \frac{10\sqrt{5}}{15} = \frac{2\sqrt{5}}{3}$$

Tutorial 3: Surds

Simplify, without a calculator:

(1) $\left(\sqrt{12}+\sqrt{3}\right)^2$

(2) $\dfrac{3\sqrt{12}-\sqrt{27}}{7\sqrt{3}+\sqrt{75}}$

(3) $\left(\dfrac{1}{\sqrt{12}+\sqrt{3}}-\dfrac{1}{\sqrt{12}+3\sqrt{3}}\right)^{-1}$

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Tutorial 3 Example 1: Suggested solution

Simplify, without a calculator:

(1) $\left(\sqrt{12}+\sqrt{3}\right)^2$

$$=12+2\sqrt{12}\sqrt{3}+3$$

$$=15+2\sqrt{36}$$

$$=15+2\times 6=27$$

Easier option?

Tutorial 3 Example 2: Suggested solution

(2) Simplify without a calculator

$$\frac{3\sqrt{12}-\sqrt{27}}{7\sqrt{3}+\sqrt{75}}$$

$$=\frac{6\sqrt{3}-3\sqrt{3}}{7\sqrt{3}+5\sqrt{3}}$$

$$=\frac{3\sqrt{3}}{12\sqrt{3}}=1/4$$

Tutorial 3 Example 3: Suggested solution

(3) Simplify without calculator

$$\left(\frac{1}{\sqrt{12}+\sqrt{3}}-\frac{1}{\sqrt{12}+3\sqrt{3}}\right)^{-1}$$

$$=\left(\frac{1}{2\sqrt{3}+\sqrt{3}}-\frac{1}{2\sqrt{3}+3\sqrt{3}}\right)^{-1}$$

$$=\left(\frac{1}{3\sqrt{3}}-\frac{1}{5\sqrt{3}}\right)^{-1}$$

$$=\left(\frac{5-3}{15\sqrt{3}}\right)^{-1}=\frac{15\sqrt{3}}{2}$$

Lesson 4

Logarithms

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Definition Logarithms

$$\text{If } x = a^y \ \text{ then } y = \log_a x$$

Exponential form **Logarithmic form**

Note:

- $a > 0$
- $a \neq 1$ (Trivial)
- $x > 0$

$\log_a(\text{zero})$ and $\log_a(\text{negative number})$

Both undefined

Example

$x = 2^y$ or $y = \log_2 x$

Changing from Exponential form to Logarithmic form and visa verse

1) From $\log_2 8 = x$

to $2^x = 8 = 2^3$

Basis the same

$\Rightarrow$ Exponents the same

$\therefore x = 3$

2) From $\log_2 p = 5$

to $p = 2^5$

$\therefore p = 32$

3) From $\log_b 25 = -2$

to $b^{-2} = 25 = \left(\frac{1}{5}\right)^{-2}$

Exponents the same

$\Rightarrow$ Basis the same

$\therefore b = \frac{1}{5}$

General Remarks

- $\log_m m = 1$ if $m > 0$ and $m \neq 1$ $\left(\because m^1 = m\right)$
- $\log_t 1 = 0$ if $t > 0$ and $t \neq 1$ $\left(\because t^0 = 1\right)$
- $\log_{10} x = \log x$

No base indicated $\Rightarrow$ That the base is 10.

- $\log 1 = 0,$ $\log 10 = 1,$ $\log 100 = 2$
- $\log 0{,}1 = \log 10^{-1} = -1,$ $\log 0{,}01 = -2$

Logarithmic Laws

Let $A, B \in \mathbb{R}^+$

Law 1: $\log_a AB = \log_a A + \log_a B$

Law 2: $\log_a A/B = \log_a A - \log_a B$

Law 3: $\log_a P^r = r \log_a P$

Law 4: $\log_a P = \dfrac{\log_s P}{\log_s a}$

Change Basis

Applications of Logarithmic Law 1

Law 1: $\log_a AB = \log_a A + \log_a B$

$$\log 5 + \log 2 = \log 10 = 1$$

$$\log 200 = \log 2 + \log 100 = \log 2 + 2$$

Applications of Logarithmic Law 2

Law 2: $\log_a A/B = \log_a A - \log_a B$

$$\log 2 + \log 3 - \log 6 = \log \frac{2\times 3}{6} = \log 1 = 0$$

Applications of Logarithmic Law 3

Law 3: $\log_a P^r = r \log_a P$

$$2\log 5 + 3\log 2 - \log 2 = \log \frac{5^2 \times 2^3}{2} = \log 1000 = 3$$

$$120 = 3^x \Rightarrow \log 120 = \log 3^x$$

$$\therefore x \log 3 = \log 120$$

$$\therefore x = \frac{\log 120}{\log 3} \simeq 4.3578 \ (\text{Use Calculator})$$

Application of Logarithmic Law 4

Law 4: $$\log_a P = \frac{\log_s P}{\log_s a}$$

$$\log_{25} 125 = \frac{\log 125}{\log 25} = \frac{3 \log 5}{2 \log 5} = \frac{3}{2}$$

$$\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{3 \log_2 2}{2 \log_2 2} = \frac{3}{2}$$

Useful Logarithm Hint

$$\log_a a^x = x$$

$$\because \log_a a^x = \frac{\log a^x}{\log a} = \frac{x \log a}{\log a} = x$$

$$\log_3 3^4 = 4$$

$$\log_2 16 = \log_2 2^4 = 4$$

$$\log_5 {}^1\!/_{125} = \log_5 5^{-3} = -3$$

$$\log_{1/2} 2^4 = \log_{1/2} \left({}^1\!/_2 \right)^{-4} = -4$$

Simplification of Logarithmic expressions without using a calculator

1) $\log_2 8 + \log_3 27 - \log_7 7 = 3 + 3 - 1 = 5$

2) $2\log 60 - 2\log 2 - 2\log 3$

$$= \log \frac{60^2}{2^2 \times 3^2} = \log \frac{3600}{36} = \log 100 = 2$$

3) $\log_{0,2} 125$

$$= \frac{\log 125}{\log 0,2} = \frac{\log 5^3}{\log 5^{-1}} = -3$$

Simplification of Logarithmic expressions without using a calculator

4) $\log_8 16 + 2\log_3 \tfrac{1}{9} - \log_3 1 - \log_8 0,25$

$$= \frac{4\log 2}{3\log 2} + \frac{(2)(-2)\log 3}{\log 3} - 0 - \frac{\log \tfrac{1}{4}}{\log 8} = \frac{4}{3} - 4 - \frac{(-2)\log 2}{3\log 2} = -2$$

5) $5\log_4 2 - \log_4 0,125 - 2\log_4 8$

$$= \log_4 \frac{2^5}{2^{-3}2^6} = \log_4 4 = 1$$

6) $\dfrac{\log 9 - \log 4}{\log 8 - \log 27} = \dfrac{2\log 3 - 2\log 2}{3\log 2 - 3\log 3} = \dfrac{2(\log 3 - \log 2)}{-3(\log 3 - \log 2)} = -\dfrac{2}{3}$

Tutorial 4: Logarithms

1) Evaluate, without a calculator:

(a) $\log\frac{3}{4}+2\log\frac{2}{5}-\log 12$

(b) $\log_5 125+\log_7 49 - 2\log_{12} 144$

(c) $\dfrac{\log 16 - \log 9}{2\log 2-\log 3}$

(d) $\dfrac{\log_x 125-\log_x \sqrt[3]{25}}{3\log_x \sqrt{5}}$

PAUSE DVD

- Do Tutorial 4
- Then View Solutions

2) Given ... c.

Express th... ms

of a, b and c.

(a) $\log 36$

(b) $\log 5$

(c) $\log\sqrt{\dfrac{12}{7}}$

Suggested solutions
Tutorial 4: Logarithms

1) Evaluate, without a calculator:

(a) $$\log\frac{3}{4}+2\log\frac{2}{5}-\log 12=\log\left(\frac{3}{4}\times\frac{4}{25}\times\frac{1}{12}\right)=\log\frac{1}{100}=-2$$

(b) $$\log_5 125+\log_7 49-2\log_{12}144=\frac{3\log 5}{\log 5}+\frac{2\log 7}{\log 7}-\frac{4\log 12}{\log 12}=3+2-4=1$$

(c) $$\frac{\log 16-\log 9}{2\log 2-\log 3}=\frac{4\log 2-2\log 3}{2\log 2-\log 3}=\frac{2\left(2\log 2-\log 3\right)}{\left(2\log 2-\log 3\right)}=2$$

(d) $$\frac{\log_x 125-\log_x\sqrt[3]{25}}{3\log_x\sqrt{5}}=\frac{3\log_x 5-{}^{2}\!/_{3}\log_x 5}{{}^{3}\!/_{2}\log_x 5}=\frac{{}^{7}\!/_{3}\log_x 5}{{}^{3}\!/_{2}\log_x 5}={}^{7}\!/_{3}\times{}^{2}\!/_{3}={}^{14}\!/_{9}$$

Suggested solutions
Tutorial 4: Logarithms (continued)

2) Given $\log 2 = a$, $\log 3 = b$ and $\log 7 = c$.

Express the following in terms of a, b and c.

(a) $\log 36 = \log\left(2^2 \times 3^2\right) = 2\log 2 + 2\log 3 = 2a + 2b$

(b) $\log 5 = \log {}^{10}\!/\!_{2} = \log 10 - \log 2 = 1 - a$

(c) $\log\sqrt{\dfrac{12}{7}} = {}^{1}\!/\!_{2}\left(\log 12 - \log 7\right) = {}^{1}\!/\!_{2}\left(\log\left(2^2.3\right) - \log 7\right)$

$= {}^{1}\!/\!_{2}\left(2\log 2 + \log 3 - \log 7\right)$

$= {}^{1}\!/\!_{2}\left(2a + b - c\right) = a + {}^{b}\!/\!_{2} - {}^{c}\!/\!_{2}$

End of the DVD on Exponents, Surds and Logarithms

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare you … hat were discussed in the …

END

- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!