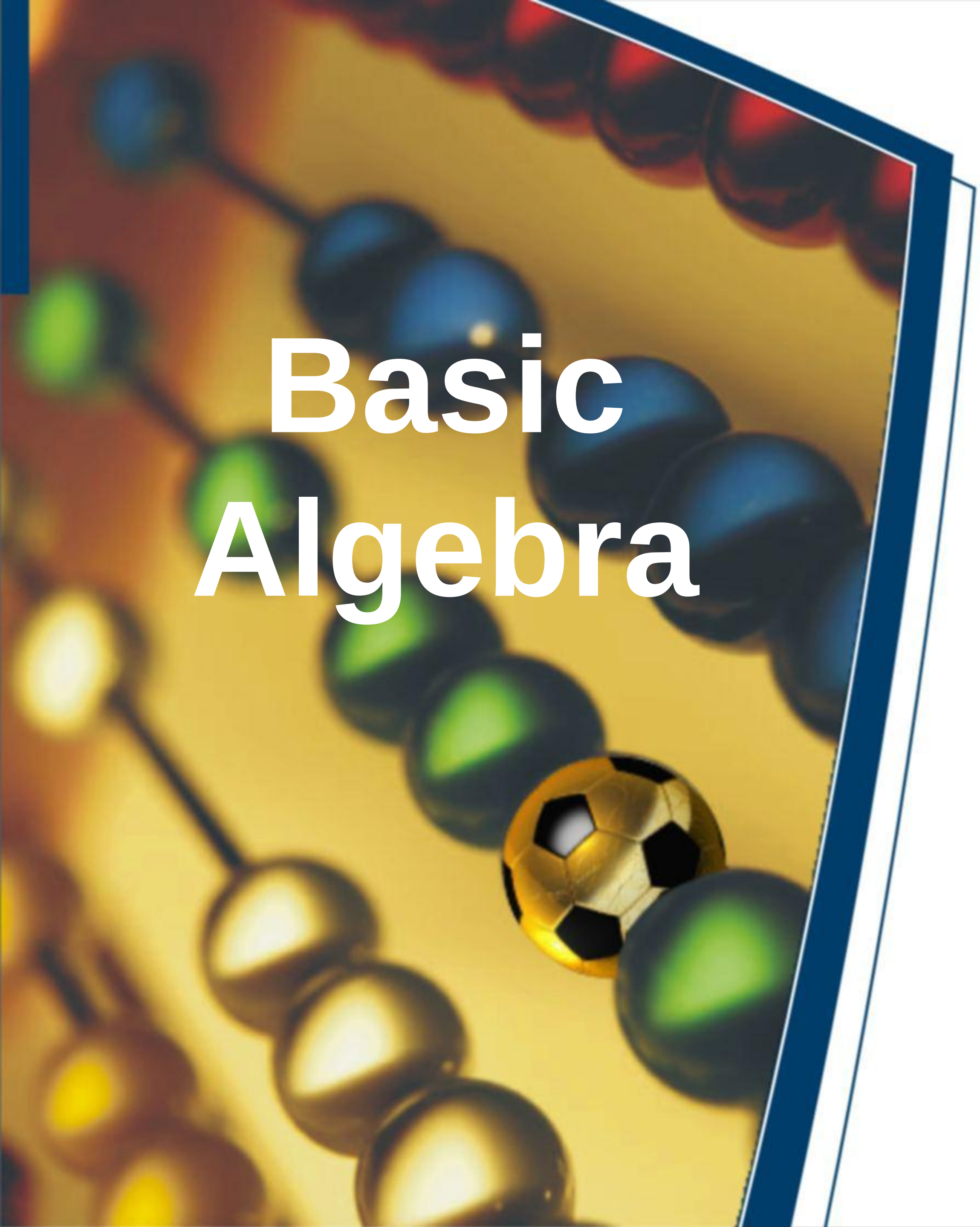

Basic Algebra

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Outcomes for this DVD

In this DVD you will:

- Revise factorization.

LESSON 1.

- Revise simplification of algebraic fractions.

LESSON 2.

- Discuss when trinomials can be factorized.

LESSON 3.

Lesson 1

Factorization

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What is Factorization?

Factorization and **Product Expansion**
are reverse processes

Product Expansion: $(a+b)(c+d)=ac+ad+bc+bd$

Use FOIL Rule

Factorize: $ac+ad+bc+bd=(a+b)(c+d)$

Product: $(a+b)(c+d+e)=ac+ad+ae+bc+bd+be$

Factorize: $ac+ad+ae+bc+bd+be=(a+b)(c+d+e)$

Methods of Factorization

- Common factor
- Difference between two squares
- Trinomials (Perfect square)
- Sum of and difference between two cubes
- Grouping

Common Factor

- Common factor(s) must occur in each term
- Must be the HCF

Highest Common Factor

Examples

1) $ab+2ac-3ad = a(b+2c-3d)$ — a is the HCF

2) $9a^3x^2-6a^2xy = 3a^2x(3ax-2y)$ — $3a^2x$ is the HCF

3) $2x(x-y)-4y(y-x)$

$= 2x(x-y)+4y(x-y)$ — Preliminary algebraic manipulation

$= 2(x-y)(x+2y)$ — $2(x-y)$ is the HCF

Difference between two squares

- Two terms only
- Terms separated by a minus
- Terms both perfect squares

$$\left(\mathbf{x}^2 - \mathbf{y}^2\right) = \left(\mathbf{x} + \mathbf{y}\right)\left(\mathbf{x} - \mathbf{y}\right)$$

Check: Use FOIL-Rule

Examples

1) $4a^2 - 9b^2 = \left(2a + 3b\right)\left(2a - 3b\right)$

2) $\dfrac{9x^2}{16} - \dfrac{y^4}{25} = \left(\dfrac{3x}{4} + \dfrac{y^2}{5}\right)\left(\dfrac{3x}{4} - \dfrac{y^2}{5}\right)$

Apply again

3) $x^4 - y^4 = \left(x^2 + y^2\right)\left(x^2 - y^2\right) = \left(x^2 + y^2\right)\left(x + y\right)\left(x - y\right)$

4) $4x^2\left(x - 2\right) + y^2(2 - x) = 4x^2\left(x - 2\right) - y^2\left(x - 2\right)$

$= \left(x - 2\right)\left(4x^2 - y^2\right)$ **Common Factor**

$= \left(x - 2\right)\left(2x + y\right)\left(2x - y\right)$ **Difference of Squares**

The Factorization of Trinomials

What is important

- Framework
- Factor Combination
- Placement of selected factor combination

Frameworks

1) $a(\ +b\ )\,a(\ +b\ )$

2) $a(\ -b\ )\,a(\ -b\ )$

3) $x(\ +y\ )\,x(\ -y\ )$

Factor Combinations

- $6 = 2\times 3 = 1\times 6$ and
- $10 = 10\times 1 = 5\times 2$

Examples

1) $6a^2 + 19ab + 10b^2 = (3a+2b)(2a+5b)$ $3\times 5 + 2\times 2 = 19$

Common factor!

2) $6a^2 - 16ab + 10b^2 = (2a-2b)(3a-5b)$ $2\times 5 + 3\times 2 = 16$

3) $6x^2 + 59xy - 10y^2 = (x+10y)(6x-y)$ $6\times 10 - 1\times 1 = 59$

4) $6x^2 - 11xy - 10y^2 = (3x+2y)(2x-5y)$ $3\times 5 - 2\times 2 = 11$

Perfect Square

Special type of Trinomial

- **First** and **last terms** are **perfect squares**
- Middle term = Twice the product of square roots of first and last terms
- Two identical factors

$$a^2 \pm 2ab + b^2 = (a \pm b)(a \pm b) = (a \pm b)^2$$

Examples

1) $4a^2 + 12ab + 9b^2 = (2a + 3b)(2a + 3b)$

$= (2a + 3b)^2$

2) $16x^4 - 40x^2y + 25y^2$

$= (4x^2 - 5y)^2$

Sum of- and difference between two Cubes

Expression consists of:

- 2 Terms only
- Terms are both cubes

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Examples

1) $27a^3 + 64b^6$

$$= (3a + 4b^2)(9a^2 - 12ab^2 + 16b^4)$$

2) $125x^9 - 64y^6$

$$= (5x^3 - 4y^2)(25x^6 + 20x^3y^2 + 16y^4)$$

Grouping

- When there are more than 3 terms in an expression
- Terms are grouped together in order to apply well-known factorization methods

Examples

$$10:-15=2:-3=-4:6$$

$$10x^5-15x^2-4x^3+6$$

$$=5x^2\left(2x^3-3\right)-2\left(2x^3-3\right) \quad =\left(2x^3-3\right)\left(5x^2-2\right)$$

OR

$$10:-4=5:-2=-15:6$$

$$10x^5-15x^2-4x^3+6=2x^3\left(5x^2-2\right)-3\left(5x^2-2\right)$$

$\Rightarrow$ Same result

Tutorial: Factorization (Part 1)

Factorize the following expressions completely:

1) $2x(x-1)-6x^2(1-x)$

2) $16x^4-81y^{12}$

3) $16p^2+4q-q^2-4$

4) $x^2+2xy+y^2-2x-2y+1$

5) $a^2(m-n)-2a(m-n)-n+m$

6) $3x^6-3y^6$

PAUSE DVD

- Do Tutorial (Part 1)
- Then View Solutions

REVISION FOR TEST

- Attempt examples on your own.
- Compare your solutions to the solutions given in the next slide.
- Repeat this procedure a few times before the next test.
- Consult text-books for additional examples.
- Remember **practice makes perfect!**

Tutorial: Factorization (Part 1) Suggested Solutions

1) $2x(x-1)-6x^2(1-x)$

$=2x(x-1)+6x^2(x-1)$

$=2x(x-1)(1+3x)$

2) $16x^4-81y^{12}$

$=(4x^2+9y^6)(4x^2-9y^6)$

$=(4x^2+9y^6)(2x+3y^3)(2x-3y^3)$

3) $16p^2+4q-q^2-4$

$=16p^2-(q^2-4q+4)$

$=16p^2-(q-2)^2$

$=(4p+q-2)(4p-q+2)$

4) $x^2+2xy+y^2-2x-2y+1$

$=(x+y)^2-2(x+y)+1$

$=(x+y-1)^2$

5) $a^2(m-n)-2a(m-n)-(m-n)$

$=(m-n)(a^2-2a-1)$ $\Delta=8$ (Not a perfect square)

6) $3x^6-3y^6$ $=3(x^6-y^6)$

$=3(x^2-y^2)(x^4+x^2y^2+y^4)$

$=3(x+y)(x-y)(x^4+x^2y^2+y^4)$

OR $=3(x^3+y^3)(x^3-y^3)$

$=3(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

Tutorial: Factorization (Part 2)

Factorize the following expressions completely:

7) $8a^3 - b^3 + 2a - b$

8) $x^3 - 2x^2 - xy + xz - 2z + 2y$

9) $4(3a+b)^2 - 9(4a-3b)^2$

10) $25y^4 - 1 - 9m^2 + 6m$

11) $2(x-1)^2 - x^2 + 1$

MAKE THIS YOUR MOTTO!

- Attempt examples on your own.
- Compare your solutions to the solutions given in the next slide.
- Repeat this procedure a few times before the next test.
- Consult text-books for additional examples.
- Remember **practice makes perfect!**

PAUSE DVD

- Do Tutorial (Part 2)
- Then View Solutions

Tutorial: Factorization (Part 2) Suggested Solutions

7) $8a^3 - b^3 + 2a - b$

$$= (2a-b)(4a^2+2ab+b^2) + (2a-b)$$
$$= (2a-b)(4a^2+2ab+b^2+1)$$

8) $x^3 - 2x^2 - xy + xz - 2z + 2y$

$$= x^2(x-2) - y(x-2) + z(x-2)$$
$$= (x-2)(x^2-y+z)$$

9) $4(3a+b)^2 - 9(4a-3b)^2$

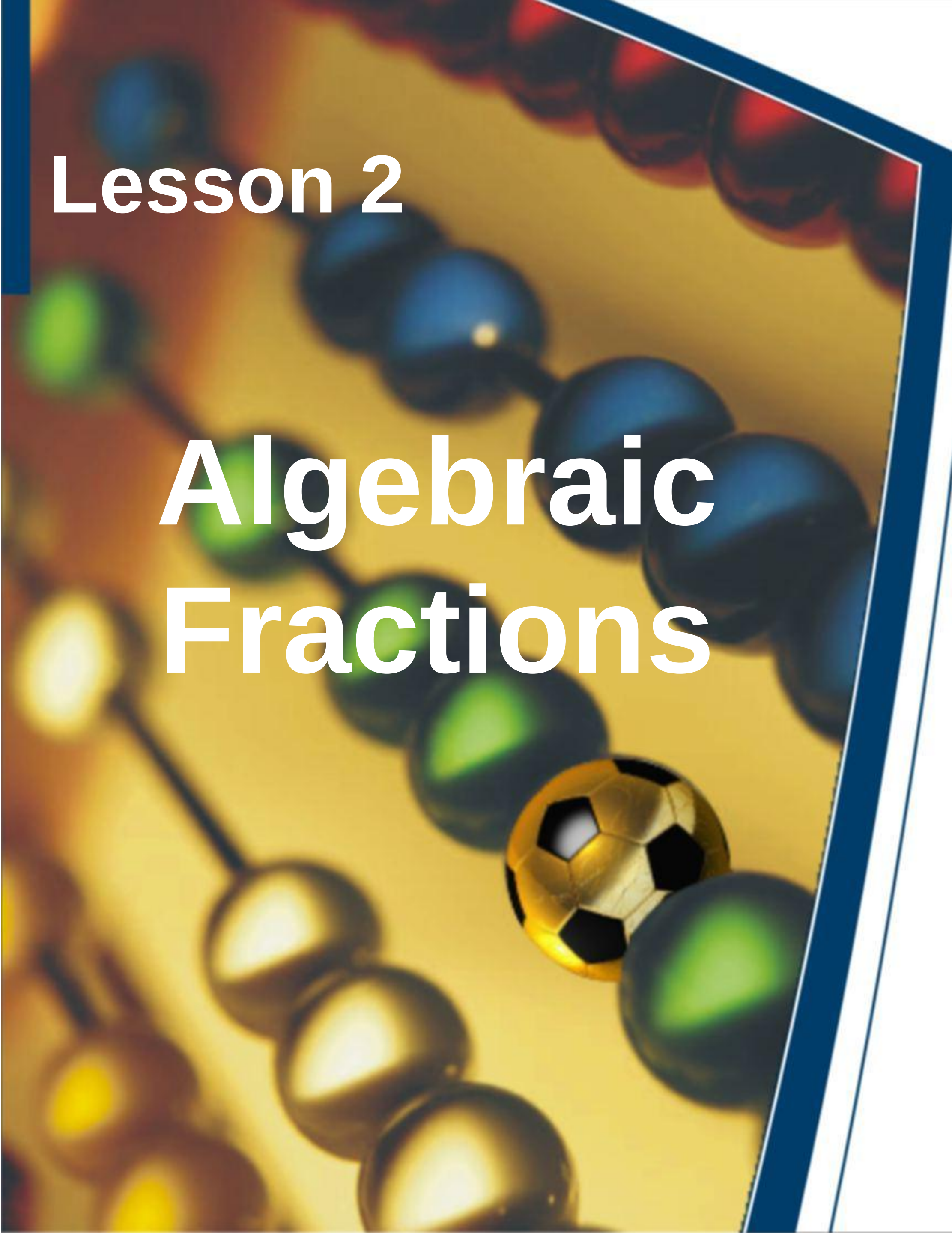
$$= [2(3a+b)+3(4a-3b)][2(3a+b)-3(4a-3b)]$$
$$= (6a+2b+12a-9b)(6a+2b-12a+9b)$$
$$= (18a-7b)(-6a+11b)$$

10) $25y^4 - 1 - 9m^2 + 6m$

$$= 25y^4 - (9m^2-6m+1)$$
$$= 25y^4 - (3m-1)^2$$
$$= (5y^2+3m-1)(5y^2-3m+1)$$

11) $2(x-1)^2 - x^2 + 1$

$$= 2(x-1)^2 - (x^2-1)$$
$$= 2(x-1)^2 - (x+1)(x-1)$$
$$= (x-1)(2x-2-x-1)$$
$$= (x-1)(x-3)$$



Lesson 2

Algebraic Fractions

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Working with Algebraic Fractions

The aim of this section is discuss **operations** with **algebraic fractions** as well as the relevant **procedures** and **restrictions**.

The following will come under discussion

- Simplification of fractions
- Multiplication and division of fractions
- Addition and subtraction of fractions
- Simplification of compound fractions

Restrictions linked to Algebraic Fractions

- If the denominator of an algebraic fraction is zero, the expression is meaningless.
- No factor in the denominator can thus be zero.
- Hence the variable(s) in the denominator cannot have certain values.

Examples

1) $\dfrac{1}{x-3}+\dfrac{4}{x+5}$ $\Rightarrow$ Restrictions: $x \neq 3$ and $x \neq -5$

2) $\dfrac{x^2-3x}{x^2-4}=\dfrac{x(x-3)}{(x+2)(x-2)}$ $\Rightarrow$ Restrictions: $x \neq 2$ and $x \neq -2$

Use of Brackets

The following information about the use of brackets is helpful when working with algebraic expressions:

$$1.)\ x + y = y + x$$

$$3.)\ x - y = -(y - x)$$

$$2.)\ -x - y = -(x + y)$$

$$4.)\ -x + y = -(x - y)$$

$$5.)\ (x - y)^2 = (y - x)^2$$

$$6.)\ (y - x)^3 = -(x - y)^3$$

$$7.)\ (y - x)^4 = (x - y)^4$$

$$8.)\ (y - x)^5 = -(x - y)^5$$

Simplification of Fractions

- Factorize both denominator and numerator
- Divide both by the same factor (s) or cancel common factors
- Only non-zero factors can be cancelled!!

Example 1

$$\frac{a^2 - 25b^2}{-2a - 10b} = \frac{(a+5b)(a-5b)}{-2(a+5b)}$$

Factorise both numerator and denominator.

$$= \frac{a-5b}{-2}$$

Cancel common factor (s).

$$= -\frac{a-5b}{2}$$

Preferred format.

Simplification of Fractions
More examples

Example 2

$$\frac{x^2 - x - 6}{9 - 3x}$$

$$= \frac{(x-3)(x+2)}{-3(x-3)}$$

Factorise both numerator and denominator.

$$= -\frac{x+2}{3}$$

Cancel common factor and write in preferred form.

Note that we assume that $x \neq 3$

Simplification of Fractions
More examples

Example 3

$$\frac{ax-b+x-ab}{ax^2-abx}$$

$$=\frac{(ax-ab)+(x-b)}{ax(a-b)}$$

Factorisation Stage 1.

$$=\frac{a(x-b)+(x-b)}{ax(x-b)}$$

Factorisation Stage 2.

$$=\frac{(x-b)(a+1)}{ax(x-b)}$$

Factorisation Final Stage.

$$=\frac{a+1}{ax}$$

Simplification.
Cancel common factors.

Multiplication and Division of Fractions

Step 1:

- Factorize all numerators and denominators.

Step 2:

- Cancel common factors.

Remember:

- When dividing with a fraction – rather multiply with the reciprocal.

Multiplication and Division of Fractions Examples

Example 1

$$\frac{ax-3a}{x^2-4}\times\frac{x^2-2x}{2x^2-5x-3}$$

$$=\frac{a(x-3)}{(x+2)(x-2)}\times\frac{x(x-2)}{(2x+1)(x-3)}$$

Factorise all numerators and denominators.

$$=\frac{a(x-3)}{(x+2)(x-2)}\times\frac{x(x-2)}{(2x+1)(x-3)}$$

Cancel common factors.

$$=\frac{ax}{(x+2)(2x+1)}$$

Simplified result.

Multiplication and Division of Fractions
More Examples

Example 2

$$\frac{x^2-12x+32}{8x}\div\frac{x^2-8x+16}{x^2-8x}\times\frac{x^2-4x}{x-8}$$

- Division to multiplication.
- Factorise all numerators and denominators.

$$=\frac{(x-4)(x-8)}{8x}\times\frac{x(x-8)}{(x-4)(x-4)}\times\frac{x(x-4)}{(x-8)}$$

$$=\frac{\cancel{(x-4)}\cancel{(x-8)}}{8\cancel{x}}\times\frac{\cancel{x}(x-8)}{\cancel{(x-4)}\cancel{(x-4)}}\times\frac{x\cancel{(x-4)}}{\cancel{(x-8)}}$$

Cancel common factors.

$$=\frac{x(x-8)}{8}$$

Simplified result

Multiplication and Division of Fractions More Examples

Example 3

$$\frac{a}{4b} \div \left[\frac{2a^2 - 2a}{a^2 + 6a + 5} \times \frac{(a+5) - b(5+a)}{4ab - 4b} \right]$$

$$= \frac{a}{4b} \div \left[\frac{2a(a-1)}{(a+5)(a+1)} \times \frac{(a+5)(1-b)}{4b(a-1)} \right]$$

Factorise all numerators and denominators.

$$= \frac{a}{4b} \times \frac{(a+5)(a+1)}{2a(a-1)} \times \frac{4b(a-1)}{(a+5)(1-b)}$$

Division to multiplication

Cancel common factors.

$$= \frac{a+1}{2(1-b)}$$

Simplified result.

Addition and Subtraction of Fractions

- Factorize denominators where necessary.
- Determine **LCM** of the denominators.
- Write all the fractions as one fraction with LCM as denominator.
- Simplify fraction as done previously (If required).

Addition and Subtraction Examples

Example 1

$$\frac{x-2}{2}-\frac{x+1}{6}$$

Lowest Common Multiple

LCM(2,6) = 6

$$=\frac{3(x-2)-(x+1)}{6}$$

Write as single fraction with LCM as denominator.

$$=\frac{3x-6-x-1}{6}$$

Simplify fraction

$$=\frac{2x-7}{6}$$

Addition and Subtraction
More Examples

Example 2

$$1 + x - \frac{2x-1}{2}$$

$$= \frac{2(1+x)-(2x-1)}{2}$$

Write as single fraction with LCM as denominator.

$$= \frac{2+2x-2x+1}{2}$$

Simplify fraction

$$= \frac{3}{2}$$

Addition and Subtraction
More Examples

Example 3

$$\frac{2ab}{a^2-b^2}+\frac{a}{a+b}+\frac{b}{b-a}+1$$

Factorize denominators and determine LCM.

$$=\frac{2ab}{(a+b)(a-b)}+\frac{a}{a+b}-\frac{b}{a-b}+1$$

Write as single fraction with LCM as denominator.

$$=\frac{2ab+a(a-b)-b(a+b)+(a+b)(a-b)}{(a+b)(a-b)}$$

Simplify fraction

$$=\frac{2ab+a^2-ab-ab-b^2+a^2-b^2}{(a+b)(a-b)}$$

$$=\frac{2a^2-2b^2}{(a+b)(a-b)} \quad =\frac{2(a^2-b^2)}{(a+b)(a-b)} \quad =2$$

Simplification of Compound Fractions

- Compound fraction is a fraction in which numerator and/or denominator contain fractions.

- “Inside Denominators “ are those denominators which form part of fractions in the denominator and/or numerator parts of the compound fraction.

- The idea is to eliminate all these “inside denominators” i.e. to change the compound fraction into an ordinary algebraic fraction.

Method:

- Multiply numerator and denominator of the compound fraction with the LCM of all “inside denominators”.

Simplify Compound Fractions Examples

Example 1

$$\frac{\frac{k}{m}-\frac{m}{k}}{\frac{k}{m}+2+\frac{m}{k}}$$

What is LCM of all "inside denominators"?

$$=\frac{\frac{k}{m}-\frac{m}{k}}{\frac{k}{m}+2+\frac{m}{k}}\times\frac{mk}{mk}$$

Multiply numerator and denominator with this LCM

$$=\frac{k^2-m^2}{k^2+2mk+m^2} \quad =\frac{(k+m)(k-m)}{(k+m)^2}=\frac{k-m}{k+m}$$

Simplify fraction

Simplify Compound Fractions
More Examples

Example 2

$$\frac{\frac{1}{x-1}+\frac{1}{x}}{\frac{2}{x}-\frac{3}{x}-1}$$

What is LCM of all "inside denominators"?

Multiply numerator and denominator with this LCM

$$=\frac{\frac{1}{x-1}+\frac{1}{x}}{\frac{2}{x}-\frac{3}{x}-1}\times\frac{x(x-1)}{x(x-1)}$$

Simplify fraction

$$=\frac{x+(x-1)}{2(x-1)-3(x-1)-x(x-1)} \quad =\frac{2x-1}{2x-2-3x+3-x^2+x}=\frac{2x-1}{1-x^2}$$

Simplify Compound Fractions
More Examples

Example 3

$$\frac{x+1+\frac{1}{x-1}}{2x-1-\frac{1}{x-1}}$$

What is LCM of all "inside denominators"?

Multiply numerator and denominator with this LCM

$$=\frac{x+1+\frac{1}{x-1}}{2x-1-\frac{1}{x-1}}\times\frac{x-1}{x-1}$$

Simplify fraction

$$=\frac{x(x-1)+(x-1)+1}{2x(x-1)-(x-1)-1}=\frac{x^2-x+x-1+1}{2x^2-2x-x+1-1}=\frac{x^2}{2x^2-3x}=\frac{x}{2x-3}$$

Tutorial: Algebraic Fractions (Part 1)

Simplify the following fractions:

1) $$\frac{x^2-y^2}{x^2-2xy+y^2}$$

2) $$\frac{2a^2-2ab-4b^2}{a^2+2ab+b^2}\div\frac{a^2-4ab+4b^2}{a+b}+\frac{2}{2b-a}$$

3) $$\frac{x}{x-1}+\frac{x\left(1+x^2\right)}{1-x^3}-\frac{1+x}{1+x+x^2}$$

PAUSE DVD

- Do Tutorial (Part 1)
- Then View Solutions

Tutorial: Algebraic Fractions (Part 1) Suggested Solutions

Simplify:

1) $$\frac{x^2 - y^2}{x^2 - 2xy + y^2}$$

$$= \frac{(x+y)(x-y)}{(x-y)^2}$$

$$= \frac{x+y}{x-y}$$

Tutorial: Algebraic Fractions (Part 1) Suggested Solution

Simplify:

2) $\dfrac{2a^2-2ab-4b^2}{a^2+2ab+b^2} \div \dfrac{a^2-4ab+4b^2}{a+b} + \dfrac{2}{2b-a}$

$= \dfrac{2(a+b)(a-2b)}{(a+b)(a+b)} \times \dfrac{a+b}{(a-2b)(a-2b)} - \dfrac{2}{a-2b}$

$= \dfrac{2}{a-2b} - \dfrac{2}{a-2b}$

$= 0$

Tutorial: Algebraic Fractions (Part 1) Suggested Solution

Simplify:

3) $\dfrac{x}{x-1}+\dfrac{x\left(1+x^2\right)}{1-x^3}-\dfrac{1+x}{1+x+x^2}$

$$=\frac{x}{x-1}-\frac{x\left(1+x^2\right)}{(x-1)\left(x^2+x+1\right)}-\frac{x+1}{x^2+x+1}$$

$$=\frac{x\left(x^2+x+1\right)-x\left(1+x^2\right)-(x+1)(x-1)}{(x-1)\left(x^2+x+1\right)}$$

$$=\frac{x^3+x^2+x-x-x^3-x^2+1}{(x-1)\left(x^2+x+1\right)}=\frac{1}{x^3-1}$$

Tutorial: Algebraic Fractions (Part 2)

Simplify the following fractions:

4) $$\frac{2}{x^2 - x} + \frac{5}{x^2 - x^3} + \frac{3}{x^2 - 1}$$

5) $$\frac{2 + \frac{1}{a}}{\frac{1}{a^2} - 4}$$

6) $$\frac{x + \frac{1}{2}}{x - \frac{1}{2}} \div \frac{2 + \frac{1}{x}}{\frac{2x}{3} + \frac{1}{3}}$$

PAUSE DVD

- Do Tutorial (Part 2)
- Then View Solutions

Tutorial: Algebraic Fractions (Part 2) Suggested Solutions

Simplify:

4) $\dfrac{2}{x^2-x}+\dfrac{5}{x^2-x^3}+\dfrac{3}{x^2-1}$

$$=\frac{2}{x(x-1)}-\frac{2}{x^2(x-1)}+\frac{3}{(x+1)(x-1)}$$

$$=\frac{2x(x+1)-2(x+1)+3x^2}{x^2(x+1)(x-1)}$$

$$=\frac{2x^2+2x-2x-2+3x^2}{x^2(x+1)(x-1)}=\frac{5x^2-2}{x^2(x+1)(x-1)}$$

Tutorial: Algebraic Fractions (Part 2) Suggested Solutions

Simplify:

5) $$\frac{2+\frac{1}{a}}{\frac{1}{a^2}-4}$$

$$=\frac{\left(2+\frac{1}{a}\right)\times a^2}{\left(\frac{1}{a^2}-4\right)\times a^2}$$

$$=\frac{2a^2+a}{1-4a^2}=\frac{a(2a+1)}{(1-2a)(1+2a)}=\frac{a}{1-2a}$$

Tutorial: Algebraic Fractions (Part 2) Suggested Solution

Simplify:

6) $$\frac{x+\frac{1}{2}}{x-\frac{1}{2}} \div \frac{2+\frac{1}{x}}{\frac{2x}{3}+\frac{1}{3}}$$

$$=\frac{x+\frac{1}{2}}{x-\frac{1}{2}} \times \frac{\frac{2x}{3}+\frac{1}{3}}{2+\frac{1}{x}} = \frac{\left(x+\frac{1}{2}\right)\times 2}{\left(x-\frac{1}{2}\right)\times 2} \times \frac{\left(\frac{2x}{3}+\frac{1}{3}\right)\times 3x}{\left(2+\frac{1}{x}\right)\times 3x}$$

$$=\frac{(2x+1)}{(2x-1)} \times \frac{x(2x+1)}{3(2x+1)} = \frac{x(2x+1)}{3(2x-1)}$$

Lesson 3

When can a Trinomial be Factorized?

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Completion of the Square

Consider a general trinomial $ax^2 + bx + c$

Completing the square:

$a, b, c \in \mathbb{R}; a \neq 0$

$$a\left(x^2 + \frac{bx}{a} + \frac{c}{a}\right) = a\left[x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] = a\left[\left(x + \frac{b}{2a}\right) + \frac{\sqrt{\Delta}}{2a}\right]\left[\left(x + \frac{b}{2a}\right) - \frac{\sqrt{\Delta}}{2a}\right]$$

Factorization of a Trinomial: Different Scenarios

$$ax^2+bx+c=a\left[\left(x+\frac{b}{2a}\right)+\frac{\sqrt{\Delta}}{2a}\right]\left[\left(x+\frac{b}{2a}\right)-\frac{\sqrt{\Delta}}{2a}\right]$$

1) If $\Delta=0$ then ax^2+bx+c is a perfect square

and $ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2$

2) If Δ is a **perfect square** then the trinomial will have **rational roots**.

3) If Δ is **positive,** but not a perfect square, then the trinomial will have **irrational roots.**

4) If Δ is **negative** then the trinomial cannot be factorized (**imaginary roots**).

Examples

$$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right) + \frac{\sqrt{\Delta}}{2a}\right]\left[\left(x + \frac{b}{2a}\right) - \frac{\sqrt{\Delta}}{2a}\right]$$

1) In $x^2 - 12x + 36$ $\Delta = 0$ thus $x^2 - 12x + 36$ is a perfect square.

Furthermore $x^2 - 12x = 36 = (x-6)^2$

2) In $6x^2 - 11x - 10$ $\Delta = 361 = 19^2$ is a perfect square.

$\therefore$ $6x^2 - 11x - 10 = (3x+2)(2x-5)$

3) In $6x^2 - 12x - 10$ $\Delta = 384 \simeq 19.59.$

$\therefore$ Factors are irrational.

4) In $9x^2 + 12x + 16$ $\Delta = -432 < 0.$

$\therefore$ Trinomial cannot be factorized.

Tutorial: When can Trinomials be factorized?

Comment on the factorization of the following trinomials:

1) $16x^2 + 20x + 25$

2) $16x^2 - 25x + 25$

3) $16x^2 - 41x + 25$

4) $16x^2 - 9x - 25$

PAUSE DVD

- Do Tutorial
- Then View Solutions

Tutorial: When can Trinomials be factorized?
Suggested Solutions

Comment on the factorization of the following Trinomials:

1) $16x^2 + 20x + 25$

2) $16x^2 - 40x + 25$

3) $16x^2 - 41x + 25$

4) $16x^2 - 10x - 25$

1) $\Delta = 20^2 - 4 \times 16 \times 25 = -1200 < 0$

$\therefore$ Cannot be factorized

2) $\Delta = (-40)^2 - (4)(16)(25) = 0$

$\therefore$ Trinomial is a perfect square

3) $\Delta = (-41)^2 - 4 \times 16 \times 25 = 9^2$

$\therefore$ Trinomial will have rational factors

4) $\Delta = (-10)^2 - (4)(16)(-25) = 1700 \simeq 41.23$

$\therefore$ Trinomial will have irrational factors

End of the DVD on Basic Algebra

REMEMBER!

- Consult text-books for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare you … hat were discussed in the …
- Repeat this procedure until you are confident.
- Do not forget:

Practice makes perfect!

END