EUCLIDEAN GEOMETRY QUESTIONS FROM PREVIOUS YEARS' QUESTION PAPERS

NOVEMBER 2008

QUESTION 7

7.1 Complete the statements below by filling in the missing word(s) so that the statements are CORRECT:

7.1.1 The angle subtended by a chord at the centre of a circle is .... (1)

7.1.2 The angle between the tangent and a chord is .... (1)

7.1.3 The opposite angles of a cyclic quadrilateral are .... (1)

7.2 In the figure below, RDS is a tangent to circle O at D. If BC = DC and \( \hat{CDS} = 40^\circ \), calculate, with reasons, the measures of:

7.2.1 \( \hat{BDC} \) (2)

7.2.2 \( \hat{C} \) (1)

7.2.3 \( \hat{A} \) (1)

7.2.4 \( \hat{O_1} \) (2)
QUESTION 8

In the diagram below, points R, P, A, Q and T lie on a circle. RA bisects $\hat{R}$ and $AB = AQ$. RA and TQ produced meet at B.

Prove that:

8.1 $AQ$ bisects $\hat{PQB}$  

8.2 $TR = TB$  

8.3 $\hat{P} = \hat{R}P$  

[8]
QUESTION 9

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.
Let $\hat{P}_1 = x$

9.1 Why is $\hat{P}_1 = \hat{Q} = 90^\circ$? (1)

9.2 Prove that $\hat{P}_1 = \hat{S}$. (3)

9.3 Prove that SRWT is a cyclic quadrilateral. (3)

9.4 Prove that $\triangle QWR \parallel \triangle QST$. (3)

9.5 If $QW = 5\ cm$, $TW = 3\ cm$, $QR = 4\ cm$ and $WR = 2\ cm$, calculate the length of:

9.5.1 TS (3)

9.5.2 SR (3)

[16]
QUESTION 10

In the figure below, \( \triangle ABC \) has D and E on BC. BD = 6 cm and DC = 9 cm. AT : TC = 2 : 1 and AD \parallel TE.

10.1 Write down the numerical value of \( \frac{CE}{ED} \) \hspace{1cm} (1)

10.2 Show that D is the midpoint of BE. \hspace{1cm} (2)

10.3 If FD = 2 cm, calculate the length of TE. \hspace{1cm} (2)

10.4 Calculate the numerical value of:

10.4.1 \( \frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD} \) \hspace{1cm} (1)

10.4.2 \( \frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC} \) \hspace{1cm} (3)

[9]
QUESTION 6

6.1 Complete the statement below by filling in the missing word(s) so that the statement is CORRECT:

The angle subtended by a chord or arc at the centre of a circle is ... (1)

6.2 In the figure below, O is the centre of the circle and PT = PR.
Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$.

![Diagram]

6.2.1 Express $x$ in terms of $y$. (3)

6.2.2 If $TQ = TR$ and $x = 120^\circ$, calculate the measure of:

(a) $y$ (2)

(b) $\hat{R}_2$ (Hint: Draw QR) (3)
NOVEMBER 2009

QUESTION 6

6.1 Complete the statement below by filling in the missing word(s) so that the statement is CORRECT:

The angle subtended by a chord or arc at the centre of a circle is … (1)

6.2 In the figure below, O is the centre of the circle and PT = PR.
Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$.

![Diagram of a circle with points R, O, Q, P, and T.]

6.2.1 Express $x$ in terms of $y$. (3)

6.2.2 If TQ = TR and $x = 120^\circ$, calculate the measure of:

(a) $\hat{y}$ (2)

(b) $\hat{R}_2$ (Hint: Draw QR) (3)

[9]
QUESTION 7

In the figure TP and TS are tangents to the given circle. R is a point on the circumference.
Q is a point on PR such that $\hat{Q} = \hat{P}$.
SQ is drawn.
Let $\hat{P} = x$.

![Diagram of a circle with tangents TP and TS, and points Q, S, R on the circle.]

Prove that:

7.1 $\overline{TQ} \parallel \overline{SR}$

7.2 $\overline{QPTS}$ is a cyclic quadrilateral

7.3 $\overline{TQ}$ bisects $\angle SQP$
QUESTION 8

In the figure, $AQ \parallel RT$, $\frac{BQ}{QC} = \frac{3}{5}$ and $\frac{BR}{RA} = \frac{1}{2}$.

8.1 If $BT = k$, calculate $TQ$ in terms of $k$.  \hfill (3)

8.2 Hence, or otherwise, calculate the numerical value of:

8.2.1 $\frac{CP}{PK}$ \hfill (3)

8.2.2 $\frac{\text{Area } \triangle RCT}{\text{Area } \triangle ABC}$ \hfill (4)

[10]
QUESTION 9

In the accompanying figure, AB is the diameter of circle ADCB. Chords AC and BD intersect at E. EP is perpendicular to AB.

9.1 Prove that \( \triangle BPE \parallel \triangle BDA \). \hspace{1cm} (3)

9.2 Hence show that \( \frac{BP}{BD} = \frac{PE}{AD} \). \hspace{1cm} (2)

9.3 Prove that \( AB^2 = BD^2 + \frac{BD^2 \cdot PE^2}{BP^2} \). \hspace{1cm} (5) \hspace{1cm} [10]
QUESTION 8

8.1 Complete the statement:

The sum of the angles around a point is ... (1)

8.2 In the figure below, O is the centre of the circle. K, L, M and N are points on the circumference of the circle such that LM = MN. \( \angle OLN = 100^\circ \).

Determine, with reasons, the values of the following:

8.2.1 \( \angle LMN \) (3)

8.2.2 \( \angle LKM \) (3) [7]
QUESTION 9

9.1 Complete the following statement:

The angle between the tangent and the chord… \( (1) \)

9.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines.

Let \( \hat{Q} = x \).

9.2.1 Name, with reasons, THREE other angles equal to \( x \). \( (5) \)

9.2.2 Prove that APTR is a cyclic quadrilateral. \( (5) \) [11]
QUESTION 10

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle. OE || CA.

10.1 Prove, with reasons, that AE = BE. \(2\)

10.2 Prove that \(\triangle AED \parallel \triangle CEB\). \(3\)

10.3 Hence, or otherwise, show that \(AE^2 = DE \cdot CE\). \(2\)

10.4 If \(AE \cdot EB = EF \cdot EC\), show that E is the midpoint of DF. \(3\) [10]
EUCLIDEAN GEOMETRY: GRADE 12

QUESTION 11

$\triangle ABC$ is a right-angled triangle with $\angle B = 90^\circ$. D is a point on AC such that $BD \perp AC$ and E is a point on AB such that $DE \perp AB$. E and D are joined.

$AD : DC = 3 : 2$.
$AD = 15 \text{ cm}$.

11.1 Prove that $\triangle BDA \parallel \triangle CDB$. 

11.2 Calculate BD (Leave your answer in surd form). 

11.3 Calculate AE (Leave your answer in surd form).
QUESTION 6

Given: $T_{k+1} = T_k + (5 - 4k)$ where $T_1 = 3$ and $k \geq 1$

6.1 Determine the FIRST FOUR terms of the sequence. (3)

6.2 What type of sequence will this formula generate? Give a reason for your answer. (2)

[5]

QUESTION 7

In the diagram below AC is a diameter of the circle with centre O. AC and chord BD intersect at E. AB, BC and AD are also chords of the circle. OD is joined. AE $\perp$ BD.

If $\hat{C} = 33^\circ$, calculate, with reasons, the size of:

7.1 $\hat{A}_1$ (3)

7.2 $\hat{D}_2$ (2)

7.3 Show that AE bisects $\hat{DAB}$ (3) [8]
QUESTION 8

8.1 In the diagram below O is the centre of the circle. GH is a tangent to the circle at T. J and K are points on the circumference of the circle. TJ, TK and JK are joined.

Prove the theorem that states $\hat{T}_1 = \hat{TJK}$. (5)

8.2 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F. BD $\parallel$ AO. $\hat{E} = x$.

8.2.1 Write down, with reasons, THREE other angles equal to $x$. (4)

8.2.2 Determine, with reasons, $\hat{CBE}$ in terms of $x$. (3)

8.2.3 Prove that F is the midpoint of BE. (4)

8.2.4 Prove that $\triangle CBD \parallel \triangle CEB$. (2)

8.2.5 Prove that $2EF.CB = CE.BD$. (3)
QUESTION 9

In the diagram below A, B, C and D are points on the circumference of the circle. BD and AC intersect at E. Also,
EB = 8 cm,
DC = 8 cm and

If DE = x units and AB = y units, calculate x and y. [6]

QUESTION 10

In the diagram below M is the centre of the circle. FEC is a tangent to the circle at E. D is the midpoint of AB.

10.1 Prove MDCE is a cyclic quadrilateral. (3)
10.2 Prove that $MC^2 = MB^2 - DC^2 - DB^2$. (3)
10.3 Calculate CE if AB = 60 mm, ME = 40 mm and BC = 20 mm. [10]
QUESTION 8

O is the centre of the circle. AB produced and DO produced meet at C.
BC = OA and \( \angle CO = 22^\circ \).

Calculate, with reasons, \( \angle AOD \). [5]

QUESTION 9

9.1 In the figure below O is the centre of the circle and PRST is a cyclic quadrilateral.

Prove the theorem that states \( \angle PRS + \angle PTS = 180^\circ \). [5]
In the diagram below two circles intersect one another at D and B. AB is a straight line such that it intersects the circle BCD at point E. BC is a straight line such that it intersects the circle ABD at F. DE, DB and DF are joined.

\[ \hat{B} = 180^\circ - 2x \]

**FCF = FDF**

9.2.1 Calculate, with reasons, in terms of \( x \):

(a) \( \angle DEB \)

(3)

(b) \( \angle A \)

(2)

9.2.2 Hence, or otherwise, prove \( ED \parallel BC \).

(3)

[13]
QUESTION 8

8.1 In the diagram below, O is the centre of the circle. PQ is a tangent to the circle at A. B and C are points on the circumference of the circle. AB, AC and BC are joined.

Prove the theorem that states $\angle CAP = \angle ABC$.

8.2 RS is a diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\vec{R}_2 = 50^\circ$.

Calculate the size of:

8.2.1 $\hat{WRS}$

8.2.2 $\hat{W}$

8.2.3 $\hat{P}_1$

8.2.4 Prove that $\hat{V}_1 = \hat{P}_1S$. [15]
EUCLIDEAN GEOMETRY: GRADE 12

QUESTION 9
AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.

If the radius is 10 cm and AC = 16 cm, calculate the length of ED. [5]

QUESTION 10
CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. AC || FD and FE = AB. Let \( D_4 = x \) and \( D_1 = y \).

10.1 Determine THREE other angles that are each equal to \( x \). (6)
10.2 Prove that \( \triangle BHD \parallel \triangle FED \). (5)
10.3 Hence, or otherwise, prove that \( AB \cdot BD = FD \cdot BH \). (2) [13]
QUESTION 11

ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that PC = 2AC and AD is produced to Q such that DQ = 2AD.

11.1 Show that E is the midpoint of AD. 

11.2 Prove PQ || FE. 

11.3 If PQ is 60 cm, calculate the length of FE. 

[10]
QUESTION 8

In the diagram below, AM is the diameter of the bigger circle AMP. RPS is a common tangent to both circles at P. APB and MPN are straight lines.

8.1 State the size of \( \hat{P}_1 \).  

8.2 Hence, show that BN is the diameter of the smaller circle.  

8.3 If \( \hat{M}_1 = 70^\circ \) calculate the size of each of the following angles:

8.3.1 \( \hat{A} \)  

8.3.2 \( \hat{P}_6 \)  

8.3.3 \( \hat{B} \)
QUESTION 9

In the diagram below, O is the centre of the circle with diameter RK. 
PS ⊥ RK. 
RK intersects PS at T.

9.1 If PS = 4x, write down the length of ST in terms of x. (1)

9.2 Prove that ΔRST || ΔPKT. (3)

9.3 If it is further given that TK = x and RT = 320 mm, calculate the value of x. (3) [7]
QUESTION 10

In \( \triangle PQW \), \( S \) is a point on \( PW \) and \( R \) is a point on \( QW \) such that \( SR \parallel PQ \).
\( T \) is a point on \( QW \) such that \( ST \parallel PR \).
\( RT = 6 \text{ cm} \)
\( WS: SP = 3:2 \)

Calculate:

10.1 \( WT \)  

10.2 \( WQ \)
QUESTION 10

In \( \triangle PQW \), \( S \) is a point on \( PW \) and \( R \) is a point on \( QW \) such that \( SR \parallel PQ \). 
\( T \) is a point on \( QW \) such that \( ST \parallel PR \). 
\( RT = 6 \text{ cm} \) 
\( WS : SP = 3 : 2 \)

![Diagram of \( \triangle PQW \) with points \( S \) and \( T \)]

Calculate:

10.1 \( WT \) \hspace{1cm} (3)
10.2 \( WQ \) \hspace{1cm} (4)

QUESTION 11

11.1 In the diagram below, \( O \) is the centre of the circle. \( PSRT \) is a cyclic quadrilateral. 
Prove the theorem that states \( \measuredangle PTR + \measuredangle PSR = 180^\circ \).

![Diagram of a circle with points \( P, S, T, R, O \)]

(6)
11.2 In the diagram below, \( O \) is the centre of the circle. \( AB \) is a diameter of the circle. Chord \( CF \) produced meets chord \( EB \) produced at \( D \). Chord \( EC \) is parallel to chord \( BF \). 
\( CO \) and \( AC \) are joined. 
Let \( \hat{O}_1 = 2x \)

11.2.1 Determine, in terms of \( x \), the size of \( \hat{F}_1 \). 
11.2.2 Prove that \( DF = BD \). 
11.2.3 Show that \( \hat{C}_1 = \hat{C}_2 \). 
11.2.4 If \( DF = 5 \) cm and \( OA = 6 \) cm, calculate \( \frac{\text{area } \triangle BFD}{\text{area } \triangle AOC} \).
QUESTION 7

7.1 If in \( \triangle LMN \) and \( \triangle FGH \) it is given that \( \hat{L} = \hat{F} \) and \( \hat{M} = \hat{G} \), prove the theorem that states \( \frac{LM}{FG} = \frac{LN}{FH} \).

7.2 In the diagram below, \( \triangle VRK \) has \( P \) on \( VR \) and \( T \) on \( VK \) such that \( PT \parallel RK \). VT = 4 units, PR = 9 units, TK = 6 units and VP = 2x – 10 units.

Calculate the value of \( x \).
QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord is equal ...

8.2 In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.

\[ \angle MPR = 75^\circ \]
\[ \angle PQT = 29^\circ \]
\[ \angle QTR = 34^\circ \]

Let \( TPW = a \), \( RPT = b \), \( MPQ = c \) and \( RTU = d \), calculate the values of \( a, b, c \) and \( d \).
QUESTION 9

O is the centre of the circle \( OAKB \).
AK produced intersects circle AOBT at T.
\( ACB = x \)

9.1 Prove that \( \hat{T} = 180^\circ - 2x \). (3)

9.2 Prove \( AC \parallel KB \). (5)

9.3 Prove \( \triangle BKT \parallel \triangle CAT \). (3)

9.4 If AK : KT = 5 : 2, determine the value of \( \frac{AC}{KB} \). (3)[14]
QUESTION 10

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. CM : MD = 4 : 9 and AB = 24 units.

10.1 Determine an expression for DC in terms of $x$ if CM = 4$x$ units. (1)
10.2 Determine an expression for OM in terms of $x$. (2)
10.3 Hence, or otherwise, calculate the length of the radius. (4) [7]
QUESTION 8

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K. OUV = 48° and KTU = 120°.

Calculate, with reasons, the sizes of the following angles:

8.1 \( \hat{V} \)  
8.2 \( \hat{KOU} \)  
8.3 \( \hat{U_2} \)  
8.4 \( \hat{K_1} \)  
8.5 \( \hat{K_2} \)  

[10]
QUESTION 9

9.1 Use the diagram below to prove the theorem which states that if VW \parallel YZ then \( \frac{XV}{VY} = \frac{XW}{WZ} \).

\[ \text{(6)} \]

9.2 In \( \triangle PQR \) below, B lies on PR such that 2PB = BR. A lies on PQ such that PA : PQ = 3 : 8. BC is drawn parallel to AR.

\[ \text{(2)} \]

9.2.1 Write down the value of \( \frac{\text{area of } \triangle PRA}{\text{area of } \triangle QRA} \).

9.2.2 Calculate the value of the ratio \( \frac{BD}{BQ} \). Show all working to support your answer.

\[ \text{[13]} \]
QUESTION 10

In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F.

AG = GD. \( \hat{E}_1 = x \).

10.1 Write down, with reasons, FOUR other angles each equal to \( x \). (8)
10.2 Prove that \( BE \cdot DE = AE \cdot FE \) (7)
10.3 Prove that \( \hat{B}_1 = \hat{D}_1 \) (4)
QUESTION 9

In \( \triangle ADC \), \( E \) is a point on \( AD \) and \( B \) is a point on \( AC \) such that \( EB \parallel DC \). \( F \) is a point on \( AD \) such that \( FB \parallel EC \). It is also given that \( AB = 2BC \).

9.1 Determine the value of \( \frac{AF}{FE} \) \( \ldots \) \( \frac{AF}{FE} \)

9.2 Calculate the length of \( ED \) if \( AF = 8 \text{ cm} \). \( \ldots \) \( \text{cm} \)
EUCLIDEAN GEOMETRY: GRADE 12

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

\[ \hat{E}_1 = 32^\circ \text{ and } \hat{E}_3 = 56^\circ. \]

Calculate, with reasons, the values of:

10.1 \[ \hat{E}_2 \] .......................... (2)

10.2 \[ \text{EBC} \] .......................... (3)

10.3 \[ \hat{F} \] .......................... (4)

[9]

QUESTION 11

In the diagram below, O is the centre of the circle and OB is perpendicular to the chord AC.

Prove, using Euclidean geometry methods, the theorem that states \( AB = BC \).
QUESTION 12

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W. WO produced meets KT at E.

Let \( \hat{W}_1 = x \)

12.1 Determine FOUR other angles, each equal to \( x \). (8)

12.2 Prove that \( \hat{T} = 90^\circ - x \). (3)

12.3 Prove that KE = ET. (3)

12.4 Prove that KE = OE.WE (6)

[20]